THE DOWNSIDE OF EXTERNAL-DEBT REDUCTION

Paul Beckerman*

*Independent consultant [paul.beckerman@gmail.com]. The original version of this paper was presented in an UNCTAD conference on external debt in Buenos Aires, in November 2006. An Excel workbook with a simulation exercise using the analytical framework discussed here is available on request from the writer.
Imagine that you are an economic-policy analyst in a small developing economy. You arrive in work one Monday (or Sunday) morning, and you are summoned....

The Council of Ministers has heard that many other economies have received debt reduction, implying a substantial reduction in the debt-service burden.

The Council of Ministers would like you to carry out a quantitative analysis to help answer the question, “Should we seek debt reduction? Would it be in our interest?”

You return to your cubicle and you think about it: How should I approach the problem? After some thought, you decide to proceed as follows:
You will carry out a macroeconomic projection exercise, with two basic scenarios.

The first scenario would maintain the debt-GDP ratio where it is. The second scenario would reduce the initial debt stock and then maintain the debt-GDP ratio at the new, lower level.

The projected balance of payments and fiscal accounts would show lower debt service in the second scenario.

However, the projected balance of payments and fiscal accounts would have to have lower external government borrowing, so as to maintain the lower debt-GDP ratio.

Imports would then have to be lower, and the reduced government deficit implies that tax revenue must be higher or /and expenditure must be lower.

Real GDP growth might have to be lower, as a consequence.
If real GDP growth would have to be lower in the reduced-debt scenario, that would be an argument against seeking debt reduction.

One of the basic assumptions underlying the debt-reduction activity over the past two decades has been that debt reduction, and the consequent reduction in debt service, would free financial resources for development expenditure.

While debt reduction might work this way, the unfavorable effects of reduced financial inflows following debt reduction could offset, perhaps more than offset, the favorable effects of reduced debt service.

(This would be a case of the “law of unintended consequences,” alas, all too familiar in economic policy-making.)
This paper sets out a macroeconomic-programming framework that our analyst could use to examine the question. It then uses the exercise illustratively to show the comparative consequences of reducing and not reducing an economy’s external debt.

The basic purpose of this presentation is to show the macroeconomic-programming framework and how it works.

The numbers used in the illustrative projection exercise are intended to be typical for developing economies, but of course the results for specific economies might differ.

The paper concludes with a brief discussion of the concept of “debt sustainability.”
Equations [(1)-(20)], presented below, are a basic multiannual macroeconomic projection exercise, encompassing the fiscal, external, national, and monetary accounts.

This exercise’s original purpose was to set out the basic macroeconomic system structure in a simplified way, in the form of equations – to show how the different pieces of a macroeconomic system “hang together”.

The idea was to make it easier for government and central-bank analysts to understand more sophisticated and detailed projection exercises, such as the World Bank’s “Revised Minimum Standard Model” (the “RMSM-X”) and the IMF’s analytical framework.

Nevertheless, the simplified exercise as presented here can be applied to construct useful rough, “stylized” spreadsheet projections for actual economies.
The essence of the approach is to program real growth rates of (a) GDP, (b) exports of goods and non-factor services, (c) government expenditure, (d) external government debt, and (e) population for each future year, and then calculate the implied growth rates of (i) per-capita non-government consumption, (ii) non-government external debt and (iii) government internal debt.

Inadequate (or even negative) per-capita non-government consumption growth and/or very rapid growth of non-government external debt and government internal debt would imply that, taken together, the programmed growth rates of GDP, government expenditure, and government external debt would not be feasible.
In this simplified programming exercise, the macroeconomic aggregates – i.e., GDP and the various components of expenditure – are all taken to be in real terms.

(If applied in an actual projection exercise, it is generally useful to express them in constant dollars, i.e., dollar values at a base year’s prices and exchange rate.)

This version of the exercise therefore abstracts from such important variables as the price level, the exchange rate, and the terms of trade. It does, however, encompass (a) the GDP-capital formation relationship, (b) the national accounts, (c) the general-government accounts, (d) the external accounts, and (e) the monetary accounts.
Define the following variables:

\[ Y \text{ real GDP;} \]
\[ I \text{ total real capital formation (including government capital formation);} \]
\[ K' \text{ year-end real capital stock;} \]
\[ T \text{ real government revenue;} \]
\[ Z \text{ real external transfers to government;} \]
\[ G \text{ real government consumption expenditure;} \]
\[ J \text{ real government capital formation;} \]
\[ H \text{ real government internal transfers and subsidies;} \]
\[ X \text{ real exports of goods and non-factor services;} \]
\[ Q \text{ real imports of goods and non-factor services;} \]
\[ F \text{ net real current-account inflows excluding net exports, real external transfers to government, and net interest due;} \]
\[ W \text{ net real non-debt capital inflows;} \]

(continues)
Define the following variables (continued):

- $E'$: real year-end stock of government external debt;
- $L'$: real year-end stock of central-bank external liabilities;
- $A'$: real year-end stock of central-bank external assets;
- $U'$: real year-end stock of non-government external debt;
- $D'$: real year-end stock of government internal debt;
- $M'$: real year-end money stock;
- $B'$: real year-end monetary base;
- $V'$: real year-end central-bank net internal assets;
- $S$: real non-government saving, and
- $P$: real government primary surplus.

The primes following the variables $E'$, $L'$, $A'$, $U'$, and $D'$ indicate that they are year-end stock values. The variables $E$, $L$, $A$, $U$, and $D$ (that is, without the prime) are the year-average values corresponding to the year-end values:

\[ E = \frac{[E'_{-1} + E']}{2} = E'_{-1} + (\Delta E'/2), \] where $\Delta E' = E' - E'_{-1};$

likewise for the year-average values $L$, $A$, $U$ and $D$. 
The following variables are projection assumptions:

\( v \) incremental capital-output ratio, net of capital depreciation;

\( v' \) capital-output ratio;

\( d \) depreciation rate of the capital stock;

\( r_U \) interest rate on non-government external debt;

\( r_D \) interest rate on government internal debt;

\( r_E \) interest rate on government external debt;

\( r_L \) interest rate on central-bank external liabilities;

\( r_A \) interest rate on central-bank external assets;

\( m \) marginal money multiplier;

\( t \) elasticity of revenue \( T \) with respect to \( Y \);

\( q \) elasticity of imports \( Q \) with respect to \( Y \);

\( f \) ratio of \( F \) to \( Y \); and

\( w \) ratio of \( W \) to \( Y \).
In general, the growth rate of any variable “x” is given by 
\((x/x_{-1})-1\). The values of the following variables are 
programming assumptions:

- \(a = A'/(Q/12)\),
- \(g_Y = (Y/Y_{-1})-1;\)
- \(g_Z = (Z'/Z_{-1})-1;\)
- \(g_G = (G/G_{-1})-1;\)
- \(g_H = (H/H_{-1})-1;\)
- \(g_J = (J/J_{-1})-1;\)
- \(g_L = (L'/L'_{-1})-1;\)
- \(g_E = (E'/E'_{-1})-1; and\)
- \(g_N = (N'/N_{-1})-1.\)
The real flows of interest due on government internal and external debt and on non-government external debt are taken to be based on the year-average debt stock.

Thus, interest due on the government’s external debt is given by

\[ r_E E = r_E (E'_{-1} + E')/2 = r_E E'_{-1} + r_E \Delta E'/2, \]

where

\[ \Delta E' = E' - E'_{-1} \]

\[ = E'_{-1} g_E. \]
The first equation of the projection system gives gross fixed capital formation in year $t$ as a function of

(i) the programmed growth rate “$g(Y)$” in the following year, (ii) the incremental capital-output ratio for capital formation carried out in year $t$, and (iii) the capital-depreciation rate $d$,

$$I = Y \nu [g_{Y(+1)} + d]. \quad [1]$$

This is because GDP growth in year $t+1$ is given by

$$\Delta Y_{+1}/Y = (I/Y)/\nu - d (K'_{-1}/Y)/\nu'$$

$$= (I/Y)/\nu + d,$$

since by definition $K'_{-1}/Y = \nu'$. 

Next, government revenue (excluding external transfers) is given by

\[ T = T_{-1} (1 + g_Y)^t, \quad [2] \]

where “t” is a policy-programming assumption.
External transfers to government are given by

\[ Z = Z_{-1} (1 + g_Z). \]  

[3]
The non-interest government-expenditure flows are based on the programming assumptions for “$g_G$”, “$g_J$” and “$g_H$”.

These are government consumption expenditure,

$$G = G_{-1} (1 + g_G); \quad [4]$$

government expenditure on gross fixed capital formation,

$$J = J_{-1} (1 + g_J); \quad [5]$$

and government internal transfers and subsidies,

$$H = H_{-1} (1 + g_H). \quad [6]$$
Exports and imports of goods and non-factor services follow from the assumptions for \( g_X \) and \( q \).

**Exports of goods and non-factor services** are given by

\[
X = X_{-1} (1 + g_X)
\]  \[7\]

(as explained below, the growth rate of exports should **not** be linked positively to real-GDP growth),

and **imports of goods and non-factor services** by

\[
Q = Q_{-1} (1 + g_Y)q.
\]  \[8\]
Non-government consumption is then determined residually so as to be consistent with the projections of G, I, X and Q.

\[ C = Y - [(G + J) + (I - J) + (X - Q)] \]

\[ = Y - (G + I + X - Q). \quad [9] \]

One basic purpose of the exercise is to determine whether the growth rates \( g_Y, g_X, g_G, g_I, \) and \( g_H \) taken together constitute a feasible macroeconomic “program,” given the other assumptions and initial conditions of the economy.

The growth rate of C, or, more precisely, of per-capita non-government consumption \( (C/N) \) is one feasibility indicator. If the growth rate of \( C/N \) is too low, then the programming assumptions would be judged unfeasible from this viewpoint.
In the balance of payments, “other” current-account flows – that is, current-account flows other than net exports of goods and non-factor services, net interest, and external transfers to government, encompassing non-interest factor-service net exports and all other net unrequited transfers – are given by

$$F = f Y.$$  \[10\]
Non-debt capital account flows (including net investment, short-term non-debt financial flows, and net errors and omissions) are given by

\[ W = w Y. \]  \[11\]
One of this exercise’s basic “strategic” approaches is to program – that is, to assume – the growth rates of the government external debt and the central bank’s net external liabilities.

It is then possible to calculate both the net increase in non-government net external liabilities, $U'$, that would be required to complete the financing of the external accounts, and the net increase in the government’s net internal liabilities, $D'$, that would be required to complete the financing of the fiscal accounts.

If the growth rates of either $U'$ or $D'$ would need to be significantly higher than the respective sources of finance would be likely to be willing and able to provide – in particular, if they significantly exceed the growth rate of real GDP – then the programmed macroeconomic policy and government expenditure would presumably be unfeasible, and would need to be adjusted.
That is, applying the programmed values $g_E$ and $g_L$, the flow increases in the government’s external debt and the central bank’s external debt are given by

$$\Delta E' = g_E E'_{-1} \quad [12]$$

and

$$\Delta L' = g_L L'_{-1}. \quad [13]$$
Using the programmed value “a” (in months of imports of goods and non-factor services) the flow increase in the central bank’s external-asset stock (basically, gross international reserves) is given by

$$\Delta A' = [a \, (Q/12)] - A'_{-1}.$$  [14]
Applying the assumptions for the interest rates on government and non-government external debt to the year-average debt stocks, and the assumptions for the interest rates on central-bank external assets to the year-average asset stock, the flow increase in non-government external debt is given by

\[
\Delta U' = r_U U + r_E E + r_L L - r_A A - (X - Q + Z + F + W) - \Delta E' - \Delta L' + \Delta A'. \quad [15]
\]

This equation is simply a rearrangement of the balance-of-payments identity. That is, the sum of the current and financing accounts is

\[
[(X - Q) + r_A A - r_U U - r_E E + Z + F] + [\Delta E' + \Delta U' + W],
\]

and the central-bank financing of the balance-of-payments deficit is given by \(\Delta L' - \Delta A'\).
Equation (15) may be solved for $\Delta U'$, making use of $U = U'_{-1} + (\Delta U'/2)$:

$$\Delta U' = [r_U U'_{-1} + r_E E + r_L L - r_A A$$

$$- (X - Q + F + W)$$

$$- \Delta E' - \Delta L' + \Delta A]/[1 - (r_U/2)].$$
The government’s primary surplus can then be calculated straightforwardly:

\[ P = T + Z - (G + H + J). \]
Applying the assumptions for the interest rates on government external and internal debt to their year-average stocks, the flow increase in government internal debt is given by

$$\Delta D' = r_D D + r_E E - P - \Delta E', \quad [17]$$

since the government deficit is given by

$$\Delta D' + \Delta E' = r_D D + r_E E - P.$$  

Equation (16) can be solved for $\Delta D'$, making use of

$$D = D'_{-1} + (\Delta D'/2):$$

$$\Delta D' = [r_D D'_{-1} + r_E E - P - \Delta E']/[1 - (r_D/2)].$$
Two equations describe the monetary aggregates.

Assume that the economy’s average money holding over each year, \( M \), is given (as a percentage of GDP).

Since \( M = M'_{-1} + (\Delta M'/2) \), the **flow increase in the money supply** is given by \( \Delta M' = 2 [M - M'_{-1}] \).

The **flow increase in the monetary base** is then given by

\[
\Delta B' = \Delta M'/m, \quad [18]
\]

and the flow increase in the central bank’s net internal assets is given by

\[
\Delta V' = \Delta B' + (\Delta L' - \Delta A'). \quad [19]
\]

That is, \( \Delta V' \) is whatever amount is required to ensure that the monetary authority’s flow balance identity is satisfied, with the increase in assets \( \Delta V' \) and \( \Delta A' \) “backing” the increase in liabilities \( \Delta L' \) and \( \Delta B' \).
The system’s final equation gives the year-average population,

\[ N = N_{-1} (1 + g_N). \]  \[20\]

This is used to calculate projected per-capita values.
In summary,

for each projection year, the analyst would program the government expenditure assumptions, \( g_G \), \( g_Y \), \( g_H \), as well as \( g_Y \) and \( g_E \), the growth rates of real GDP and (real) government external debt. The equations listed can then be used to solve for the growth rate of per-capita non-government consumption,

\[
g_{C/N} = \frac{[(C/C_{-1})/(1 + g_N)] - 1}{1 + g_N};
\]

the year-end non-government external-debt stock as a percentage of GDP,

\[
U'/Y = \frac{(U'_{-1} + \Delta U')}{Y}; \text{ and}
\]

the year-end government internal-debt stock as a percentage of GDP,

\[
D'/Y = \frac{(D'_{-1} + \Delta D')}{Y}.
\]
These projected values would then be examined to determine whether they are sufficiently high, in the case of per-capita non-government consumption, or sufficiently low, in the cases of the debt ratios, to ensure the program’s feasibility.
The analytical framework described thus far can be used to help determine whether debt reduction would be in an economy’s best interest from the perspective of the fiscal accounts. The procedure could be as follows:

(1) Carry out a “base” scenario projection over (say) five years, assuming that the government will maintain its net external borrowing so that the ratio of government debt to GDP remains what it was at the end of the last historical year.

(2) Carry out a second projection, with a reduction in the external debt at the end of the last historical year, with the assumption that the government will maintain its net external borrowing so that the ratio of government debt to GDP remains at the new, lower ratio.

(3) This will probably increase the ratio of government internal debt to GDP at the end of the five-year period, compared with the base scenario.

(4) Carry out a third projection, with a lower real-GDP growth rate, such that the ratio of government internal debt to GDP at the end of the five-year period is the same as it was in the base scenario.
To the extent (per-capita real) government expenditure would have to be lower, debt reduction would not be in the economy’s best interest.
The analytical framework can also be used to help determine whether debt reduction would be in an economy’s best interest from a balance-of-payments perspective. The procedure could be as follows:

(1) Carry out a “base” scenario projection over (say) five years, assuming that the government will maintain its net external borrowing so that the ratio of government debt to GDP remains what it was at the end of the last historical year.

(2) Carry out a second projection, with a reduction in the external debt at the end of the last historical year, with the assumption that the government will maintain its net external borrowing so that the ratio of government debt to GDP remains at the new, lower ratio.

(3) This will probably increase the ratio of non-government external debt to GDP at the end of the five-year period, compared with the base scenario.

(4) Carry out a third projection, with a lower real GDP growth rate, such that the ratio of non-government external debt to GDP at the end of the five-year period is the same as it was in the base scenario.
To the extent (per-capita real) GDP would have to be lower, debt reduction would not be in the economy’s best interest.
This projection exercise can be straightforwardly adapted for spreadsheet use.

Although too simplified for detailed policy analysis, it can provide some useful order-of-magnitude indicators.
One way to apply this simplified exercise would be to program the growth rates of E and L to equal that of GDP, thus maintaining the ratios of E and L to GDP unchanged, and then determine whether the implied growth rates of U and D would be less than or greater than that of GDP.

If both turn out smaller than the growth rate of GDP, an interesting exercise may then be to test smaller and smaller growth rates of E, to determine how rapidly the government’s external-debt burden could be reduced.

For debt-reduction exercises, one obvious approach would be to reduce the starting ratio E/Y to some new level – perhaps along with policy changes affecting other variables, such as government revenue or government consumption expenditure -- and then to set future growth rates of E so as to maintain that ratio thereafter.

The resulting growth rates of U and D would then indicate whether the debt reduction – and accompanying policy changes -- would ensure “sustainability.”
For example, the growth rate of the net government internal debt \((D)\) could be programmed, and the growth rates of the net government and non-government external debt then determined so as “to close” the government and external accounts.

The advantage of programming the government external debt and solving for the growth rates of the government internal and non-government external debt is that doing so more explicitly addresses the sustainability characteristics that would result from a specified government external-debt strategy.
In this exercise, the growth rate of exports, $g(X)$, should be considered independent of the real-GDP growth rate. In particular, it is inadvisable to assume, in this or in any other projection exercise, that the real export growth rate will be linked positively to the real GDP growth rate.

Linking the export and GDP growth rates positively has the effect of assuming away the basic external-accounts constraint, i.e., that faster GDP growth will induce import growth, at least in the short and probably in the medium term, without there being any way to increase export earnings.

Over the longer term, a small commodity-exporting economy would presumably tend to grow at the same real rate as the value of its exports. In the short and medium term, however, the real growth rates of its export value and GDP could diverge.

All other things -- in particular, the terms of trade -- being equal, if export growth is less than GDP growth, external debt would tend to grow faster, while if export growth exceeds GDP growth, non-government consumption would tend to grow slower.
The net external non-government and internal government debt stocks could come out equal to or less than zero in any given year.

If the net non-government external debt fell below zero, this would indicate that the economy’s non-government sectors had acquired external assets in excess of their external liabilities.

If the net internal government debt fell below zero, this would indicate that the government had acquired assets (including deposit balances) exceeding its internal liabilities.
Some care must be taken in projecting interest rates on external debt.

To the extent the programmed real-GDP growth rate is below the projected external world interest rates, the more likely it would be that the macroeconomic-policy and government-expenditure programs would prove unsustainable, since U/Y and D/Y would tend to grow faster.
The list following gives the variables in the simplified projection exercise:

**Variables and parameters:**
- Central-bank external assets: A
- Monetary base: B
- Non-government consumption expenditure: C
- Net government domestic debt: D
- Government external debt: E
- Net current-account inflows excluding net exports, transfers to govt., net interest due: F
- Government consumption expenditure: G
- Government domestic transfers and subsidies: H
- Total gross fixed capital formation: I
- Government gross fixed capital formation: J
- Capital stock: K
- Central-bank external liabilities: L
- Broad money supply: M
- Population (millions): N
- Government primary surplus: P
- Imports of goods and non-factor services: Q
- Government interest due: R
- Non-government saving: S
- Government revenue excl. external transfers: T
- Non-government external debt: U
- Central-bank net domestic assets: V
- Net non-debt capital inflows: W
- Exports of goods and non-factor services: X
- Gross domestic product (GDP): Y
- External transfers to government: Z
The list following gives the main parameters of the projection exercise.

- \( \frac{A'}{(Q/12)} \)  
- \( C/Z \)  
- Annual physical-capital depreciation rate  
- \( F/Y \)  
- Growth rate of...  
- \( H/Y \)  
- \( J/Y \)  
- \( \frac{\Delta M'\Delta B'}{} \)  
- Elasticity of Q with respect to Y  
- Interest rate on...  
- \( S/Z \)  
- Elasticity of T with respect to Y  
- Incremental capital-output ratio (ICOR) (net of depreciation)  
- Capital-output ratio  
- \( W/Y \)
The equations of the system are as follows:

1. \[ I = Y v \{[\Delta Y(+1)/Y] + d]\],

   since \[ \Delta Y(+1)/Y = \{(I/v) - d K'(-1)/v'\}/Y \]
   \[ = \{(I/Y)/v\} - d \]

   since by definition

   \[ [K'(-1)/Y] = v'; \]

2. \[ T = T(-1) \{[1 + g(Y)] \exp(t)\}; \]

3. \[ Z = Z(-1) [1 + g(Z)] \]

4. \[ G = G(-1) [1 + g(G)] \]

5. \[ H = H(-1) [1 + g(H)] \]

6. \[ J = J(-1) [1 + g(J)] \]

7. \[ X = X(-1) [1 + g(X)] \]

continues
System equations (continued):

(8) \[ Q = Q(-1) \left\{ [1 + g(Y)] \exp(q) \right\} \]
Q Imports of goods and non-factor services

(9) \[ F = f Y \]
F Net current-account inflows excluding net exports, transfers to govt., net interest due

(10) \[ W = w Y \]
W Net non-debt capital inflows

(11) \[ C = Y - (G + I + X - Q); \]
C Non-government consumption expenditure

(12) \[ \Delta E' = g(E) E'(-1); \]
\[ \Delta E' \quad \text{Government external debt} \]

(13) \[ \Delta L' = g(L) L'(-1); \]
\[ \Delta L' \quad \text{Central-bank external liabilities} \]

(14) \[ \Delta A' = \left[ Q a / 12 \right] - A(-1); \]
\[ \Delta A' \quad \text{Central-bank external assets} \]

continues
System equations (continued):

the balance-of-payments identity,

\[ \Delta E' + \Delta U' + \Delta L' - \Delta A' = r(U) U(-1) + r(E) E(-1) \]

\[ \Delta U' \text{ Non-government external debt} \]

\[ \Delta U' = r(L) L(-1) + r(A) A(-1) + (X - Q + F + Z + W) \]

where

\[ E = \frac{[E'(-1) + E']}{2} = E'(-1) + (\Delta E')/2, \]

\[ U = \frac{[U'(-1) + U']}{2} = U'(-1) + (\Delta U')/2. \]

\[ L = \frac{[L'(-1) + L']}{2} = L'(-1) + (\Delta L')/2. \]

\[ A = \frac{[A'(-1) + A']}{2} = A'(-1) + (\Delta A')/2; \]

the government-accounts identity,

\[ \Delta E' + \Delta D' = (G - T - Z + J + H) + r(D) D'(-1) + r(E) E'(-1) \]

\[ \Delta D' \text{ Net government domestic debt} \]

\[ \Delta D' = r(D) D'(-1) + r(E) E'(-1) - P \]

where

\[ D = \frac{[D'(-1) + D']}{2} = D'(-1) + (\Delta D')/2, \]

continues
System equations (concluded):

(17) \[ P + R = T - [G + H + J + r(E) E'(-1) + r(E) E'(-1)]; \]
\[ P \quad \text{Government primary surplus} \]

(18) \[ \Delta B' = \frac{(\Delta M'/m)}{m} = \frac{2 [M - M'(-1)]}{m}, \]
\[ \Delta B' \quad \text{Monetary base} \]

where \( M/Y \) is assumed;

(19) \[ \Delta V' = \Delta B' + (\Delta L' - \Delta A') \]
\[ \Delta V' \quad \text{Central-bank net domestic assets} \]

and

(20) \[ N = N(-1) [1 + g(N)] \]
\[ N \quad \text{Population (millions)} \]

Also,

\[ S = I - (F + J) - (Q - X); \]
\[ C + S = Y - T + H - r(U) U'(-1) + r(D) D'(-1). \]
Once the exercise is carried out in real terms – in base-year dollars, for example – straightforward calculations can be carried out to present the results in any units of account considered appropriate.

Thus, for example, the national, external, fiscal and monetary accounts can all be expressed as percentages of GDP, U.S. dollars, or local currency by applying projection assumptions for the external and internal price levels and the exchange rate.
Continuing in the same way:

To express the year-t capital-formation flow in current U.S. dollars, multiply by \( e(t)/e(0) \), where \( e(t) \) is the assumed annual-average exchange rate in that year and \( e(0) \) is the base-year exchange rate.

To express the same flow in current domestic-currency units, multiply by \( [p(t)/p(0)] [1/e(0)] \), where \( p(t) \) and \( p(0) \) are the GDP deflators for year \( t \) and the base year respectively.

To express the year-end year-t international-reserve stock in current U.S. dollars, multiply by \( e'(t)/e(0) \), where \( e'(t) \) is the assumed year-end exchange rate in that year.

A unit of account that is often especially useful in exercises of this kind is the per-capita real value, obtained simply by dividing the value in question by the (year-average) population.