

Absolute versus Relative Poverty, Evasion and the Optimal General Income Tax-cum-Audit Policy

Marcelo Arbex

Department of Economics

University of Windsor

Windsor, N9B 3P4, Canada

arbex@uwindsor.ca

Enlison Mattos

São Paulo School of Economics

Getulio Vargas Foundation

São Paulo, 01332-000, Brazil

enlison.mattos@fgv.br

Christian Trudeau

Department of Economics

University of Windsor

Windsor, N9B 3P4, Canada

trudeauc@uwindsor.ca

Preliminary, please do not cite

Abstract

This paper investigates the optimal general income tax and audit policies when poverty is considered a public bad in an economy with two types of individuals whose income may not be observed. Our results depend on whether poverty is measured in absolute or in relative terms. For the first measure, we replicate the results presented in Cremer and Gahvari (1995) and Wane (2001) such as "no distortion and no auditing at the top", and undetermined marginal tax rate with random auditing for the ones declared to be poor. Three different results emerge for a relative poverty measure. First, we find that the optimal marginal income tax rate on both rich and poor individuals can be positive, negative or zero. Second, those that declare to be rich can be audited randomly, similar to their counterpart poor ones. Last, the rich should be rewarded when he has incentives to misreport his type. Otherwise, fines on the rich should be the same as taxes imposed on them. For the poor, honesty may be punished as well as rewarded.

Keywords: Absolute and relative Poverty, Tax evasion, Optimum Taxation

JEL Classification: H21; H26; I32

Introduction

Poverty is one of the most serious problems faced by developing and poor countries. Approximately 1.2 billion women and men, or around forty percent of the world's labor force cannot earn enough to keep themselves and their families out of poverty. For many governments, the goal of poverty alleviation, together with job creation, is a key element for overall development. Amid a lack of action from the government, the informal sector thrives for precisely the same reason, i.e., to alleviate poverty and create jobs. Due to a lack of employment opportunities in the formal sector and long unemployment periods, many people are forced to join the informal sector to earn a living. Informal employment is, on average, precarious, low-paid and risky (Schneider and Enste, 2000; Maloney, 2004) and income levels in the informal sector are generally low and the incidence of poverty high. Moreover, informal workers in developing countries constitute the bulk of the working poor (ILO, 2010).

Programs to combat poverty and reduce tax evasion are ultimately an information problem. First, the income of those individuals whose income is below the poverty line is rarely observed. Besides, as non-targeted groups may benefit from poverty reduction plans and have incentives to mimic the behavior of those targeted, government programs have to ensure that resources are concentrated on the poor, minimizing the leakages to the non-poor. Second, the taxpayers' true incomes (formal and informal) may not be publicly observed and the government can only obtain information on individuals' income at a cost, i.e., by conducting audits. Although informal activities can potentially improve living standards by leaving more income in peoples' hands, tax evasion might as well contribute to poverty, for it deprives governments of needed resources to invest in economic development that would benefit the weaker segments of the population. Understanding the links between informality and poverty is critical for designing policy options targeted at tax evasion and poverty reduction.¹

There is a great deal of literature on poverty and optimal taxation (Kesselman, 1971; Zeckhauser, 1971; Garfinkel, 1973; Nichols and Zeckhauser, 1982; Moffit, 2003). For instance, Wane (2001) considers poverty a "public bad" or a negative externality to society's welfare and the income tax is the only instrument used as a redistributive policy. The author shows that poor

individuals can face negative or positive marginal tax rates, and all the non-poor, except the most skillful, face a strictly positive marginal tax rate. As in Mirrlees (1971), individual's income is publicly observed but not their abilities.² This literature however is silent with respect to the case where individuals can mimic their income and their type - a common feature of the literature on tax evasion. The possibility that the government may not observe the income of the households adds an additional cost to the tax administration and must be considered in the tax design (Sandmo, 2005). Sandmo (1981) and Cremer and Gahvari (1996) study the optimal linear and general income tax, respectively, when tax evasion is introduced in the standard income tax model. In Cremer and Gahvari (1996), tax evaders can influence the probability of being caught through expenditures on concealment and tax evasion can affect the progressiveness of the tax system depending on the concealment technology. In this literature, the poverty concern of the social planner, as well as the interactions between poverty and tax evasion, are not taken into consideration. This paper intends to fill out this gap.

This paper integrates the existence of poverty as a negative externality and tax evasion into an optimal general income tax problem. We characterize the optimum income tax-cum-audit structure when the government does not have full information about households' income and the economy consists of two types of individuals: rich and poor. Both types are risk averse with identical preferences but different skills. Labor supply is endogenous and poor (unskilled) agents earn a lower wage than rich (skilled) agents. This paper distinguishes itself from earlier studies on this subject (e.g., Mirrlees, 1971; Cremer and Gahvari, 1996; Wane, 2001) by (i) integrating a model in which taxpayers can mimic both their skills and income and (ii) poverty is a bad externality.

We consider a social welfare function of a full committed government and we show that the optimum tax-cum audit policies depend on whether the poverty measure is absolute or relative. For the first measure, we replicate most of the results presented in Cremer and Gahvari (1995) and Wane (2001) such as 'no distortion and no auditing at the top', and undetermined marginal tax rate with random auditing for the ones declared to be poor. Three different results emerge for relative poverty measure. First, we find that the optimal marginal income tax rate on both rich

and poor individuals can be positive, negative or zero. There is distortion at the top as long as they can influence the welfare of the whole society through a measure of poverty and a distortion might be optimum to reduce aggregate poverty. Second, those that declare to be rich can be audited randomly, similar to their counterpart poor ones. A distortion on labor supply decision on the part of rich individuals create opportunities for tax evasion, therefore, auditing them might be optimal. Last, honesty may be punished as well as rewarded. Since skilled individuals might be very attracted to mimic their type, an optimum large fine (higher than taxes) on those truth-telling poor individuals might be called to discourage skilled individuals.

The paper is organized as follows. Section I presents our set up model where poverty and tax evasion are integrated in a general income tax model with two types of individuals and enforcement is costly. In Section II, we characterize the properties of the optimal auditing probabilities, taxes and fines policies. Section III offers concluding comments.

I. The Economy

We consider an economy with two types of agents: rich (skilled) individuals and poor (unskilled) individuals. Both types are risk averse with similar preferences but different skills. The model is built on Cremer and Gahvari (1996) and Wane (2001) integrating two aspects: taxpayers can mimic their skills and income and aggregate poverty is a bad externality. Each type weights aggregate poverty differently. Preferences are separable in consumption (the numeraire), C , and labor supply, L . These are the only two goods in this economy and preferences, in the absence of taxes, are represented by

$$\begin{aligned}
 U &= u(C_i) + v(1 - L_i) - \beta_i P(C_p, C_r) \\
 L &= Y/w_i, \quad i = p, r.
 \end{aligned}$$

where U is continuous and twice differentiable, strictly increasing in C and decreasing in L , $u(C)$ is strictly concave, $u(0) = 0$ and $u'(0) \rightarrow \infty$. Consumption and labor are nonnegative. Agents of type p (poor) earn a lower wage than those of type r (rich). Denote w_i the wage (or skills) of an agent of type $i = p, r$. The consumption of each type is given by $C_i = Y_i - T_i$, where Y_i is

the individual's income and T_i is the amount of taxes paid by type i . Note that β_i is a common knowledge information and measures the aversion to aggregate poverty of agent i , and $P(C_p, C_r)$ denotes aggregate poverty which in turn depends on the consumption levels of those individuals. Last, we assume that $dU/dC_i > 0$ for any i , i.e., the benefit in utility terms of increasing an agent's own consumption is larger than eventual decreases in the inequality measure.

Enforcement is costly and rich and poor households' incomes are observable only with an audit cost A , which is strictly increasing in the number of people audited and $A' \in (0, +\infty)$. Penalties cannot exceed an individual's income and other punishments are excluded.

In this economy, the government maximizes social welfare in a utilitarian fashion, which is the sum of the individuals' welfare. Given that individuals are concerned with aggregate poverty, its objective also embraces aggregate poverty reduction. That concern leads to an important matter, which relates to poverty measurement. We assess poverty based on a comparison of resources to needs. A household is identified as poor if his/her resources fall below a poverty threshold. There are several ways to measure poverty and we consider two main categories: absolute and relative poverty measures.³

Absolute poverty measures consider exclusively the well-being of those who are defined as poor, thereby suggesting that only the condition of the poor and his/her deprivation is important, not the overall society (Simler and Arndt, 2007). Relative poverty measures define the segment of the population that is poor in comparison with the consumption (or income) of the general population. Thus, poverty is not determined by a discrete poverty line but rather it is determined relative to the overall income of the population. The relative method specifies the poverty threshold as a cut-off value in the distribution of income or expenditure and hence it can be updated automatically for changes in living standards (Foster, 1998; Muller, 2006). The measurement of poverty is beyond the scope of this paper and questions such as, should poverty be measured using an "absolute" or a "relative" approach? or how does a policymaker choose an appropriate measure?, are not addressed here. Instead, we study the design of the optimal income taxation when either one of these two measures is exogenously chosen by the government.

We follow the literature (Wane, 2001; Kanbur, Keen and Tuomala, 1994) and consider a

consumption-based poverty measure, by comparing an individual's consumption C_i , $i = r, p$, to a poverty line C^* . The poverty line can then be interpreted as a broad measure consisting of basic consumption goods and goods that are deemed valuable to acquire (e.g. medicine, children's equipment, housing, etc.). A benefit associated with measuring poverty by consumption is that it may convey better information about actual deprivation than information on income (Pirttila and Tuomala, 2004). We address the case that only the consumption of the poor C_p can be below the poverty line and it is given by $\hat{P}(C_p, C^*)$. Therefore, poverty reduction improves social welfare.⁴

The absolute poverty line determines a poverty threshold that does not change with the standard of living of the society, i.e. it is fixed over time at $C^* = \bar{C}^*$. The main implication of this approach is that the consumption of rich individuals, C_r , does not affect the aggregate measure of poverty, i.e., $\partial \hat{P}(C_p, C^*) / \partial C_r = \hat{P}_{C_r}(C_p, C^*) = 0$ for $C_r \geq C^*$, and the poverty index is decreasing in the consumption of the poor, that is, $\hat{P}_{C_p}(C_p, C^*) < 0$, $\hat{P}_{C_p, C_p}(C_p, C^*) > 0$ and $\hat{P}(C_p, C^*) \geq 0$ for all $C_p \in [0, C^*)$.

On the other hand, a relative measure of poverty uses a poverty line that is related in some way to the general standard of living of the society. In this case, both the consumption of the poor, C_p , and the consumption of the rich, C_r , can affect the poverty line, defined as $C^* = C^*(C_p, C_r)$, and consequently the poverty measure. The consumption of the poor has two effects on the poverty measure $\hat{P}(C_p, C^*(C_p, C_r))$. The direct effect is negative and indicates that if the consumption of the poor increases the aggregate measure on poverty decreases ($\partial \hat{P}(\cdot) / \partial C_p < 0$). On the other hand, the indirect effect is positive, as an increase in C_p raises the poverty line which, given C_r , increases the poverty index, that is, $\left(\partial \hat{P}(\cdot) / \partial C^* \right) (\partial C^*(\cdot) / \partial C_p) > 0$. Since relative poverty lines are often defined as a certain fraction of some central summary statistic (a fraction of the mean or median per capita income), the direct effect is assumed to be larger than the indirect effect.⁵ This implies that the total effect is that the poverty index is decreasing in the consumption of the poor ($d\hat{P}(\cdot) / dC_p < 0$). If only the consumption of the rich increases, the relative poverty measure will increase as a result of larger income inequality, i.e. $d\hat{P}(\cdot) / dC_r = \left[\left(\partial \hat{P}(\cdot) / \partial C^*(\cdot) \right) (\partial C^*(\cdot) / \partial C_r) \right] > 0$.

For ease of notation, we consider the poverty function only as a function of the consumptions

of poor and rich individuals and write $P(C_p, C_r)$ instead of $\hat{P}(C_p, C^*(C_p, C_r))$. In the case of an absolute measure of poverty, we have $dP(\cdot)/dC_p < 0$ and $dP(\cdot)/dC_r = 0$, while $dP(\cdot)/dC_r > 0$ in the case of a relative measure.

The direct mechanism consists of four functions: $Y(\bar{w})$, $p(\bar{w})$, $T(\bar{w})$, $F(\bar{w}, Y_A)$ where \bar{w} is the reported type and Y_A is the true income, i.e., the income revealed through an auditing process. It works as follows: after the agent reports his type, \bar{w} , the tax administrator assigns the income, $Y(\bar{w})$, the probability of auditing, $p(\bar{w})$, the amount of taxes to be paid, $T(\bar{w})$, and the fines of $F(\bar{w}, Y_A)$ if s/he is audited and found to have a true income of Y_A .

In equilibrium, as in Mookherjee and Png (1989) and Cremer and Gahvari (1996), the revelation principle (truth-telling equilibrium) applies in this case. We impose the maximum possible fine to agents that are found to have an income different than the one reported. This reduces the cost of satisfying the constraints without adding any cost, as nobody cheats in equilibrium.⁶ This implies that $F(\bar{w}, Y_A) = Y_A$ for any $Y_A \neq Y_{\bar{w}}$. Therefore, we are left with the task of finding the values of $F_k = F(k, Y_k)$, the fine applied to an agent of type k that is audited but who has honestly reported his income. Therefore, if $F_k < T_k$, we are rewarding honest agents of reported type k who are audited.

Individuals can cheat in two different ways: (i) by misreporting their type and (ii) by misreporting their income. It is assumed that only the misreported income action can be detected through an audit. Since the individuals can be audited (or not) and pay fines (or taxes) two different states arise. Abilities (types) and labor supply (action) are unobservable and remain private information even after an audit. An agent's income is the product of two unobservable variables and it is not costlessly observable either but can be discovered through an audit. This implies that more complex incentive constraints must be imposed to elicit truth-telling. Given this information structure, we derive the optimal revelation mechanism and investigate the properties of optimal audit and tax structures.

Some definitions are necessary at this point. Let EU_{ik} be the expected utility of type i individual who reports (announces) to be of type k and earns income Y_k . Two possibilities emerge:

both types are telling the truth or a rich agent imitates the poor one, as follows

$$EU_{kk} = (1 - p_k)u(Y_k - T_k) + p_k u(Y_k - F_k) + v_k(Y_k) - \beta_k P(C_p^e, C_r^e), \quad k = p, r.$$

$$EU_{rp} = (1 - p_p)u(Y_p - T_p) + p_p u(Y_p - F_p) + v_r(Y_p) - \beta_r P(C_p^e, C_p^e),$$

where $v_i(Y_k) = v(1 - Y_k/w_i)$ for $i = p, r$. Notice that the aggregate poverty measure is concerned with expected consumption where $C_p^e = (1 - p_p)(Y_p - T_p) + p_p(Y_p - F_p)$ and $C_r^e = (1 - p_r)(Y_r - T_r) + p_r(Y_r - F_r)$.

Next, we define the maximum utility of an individual with skill w , who faces a audit probability of p and pays a fine equal to the maximum possible income if audited or a tax T otherwise as follows

$$V_{rr} = (1 - p_r)u(\tilde{Y}(r, p_r, T_r) - T_r) + v_r(\tilde{Y}(r, p_r, T_r)) - \beta_r P(C_p^e, \tilde{C}(r, p_r, T_r)),$$

$$V_{rp} = (1 - p_p)u(\tilde{Y}(r, p_p, T_p) - T_p) + v_r(\tilde{Y}(r, p_p, T_p)) - \beta_r P(C_p^e, \tilde{C}(r, p_p, T_p)),$$

$$V_{pp} = (1 - p_p)u(\tilde{Y}(p, p_p, T_p) - T_p) + v_p(\tilde{Y}(p, p_p, T_p)) - \beta_p P(\tilde{C}(p, p_p, T_p), C_r^e),$$

where $\tilde{Y}(k, p, T)$ corresponds to the income that an agent of type k chooses for himself given the tax policy and $\tilde{C}(k, p, T)$ is the correspondent consumption. Note that in V_{ik} we assume that the agent misreports his income, which implies that \tilde{Y} is independent of F_k . This formulation corresponds to an agent of type i that claims to be of type k but declares a different income than the one assigned to type k . Since this type of cheating is detected, we call it income misreporting. Moreover, in this specification we allow the individuals to recognize their effect on the correspondent poverty measure they should be concerned with.

We assume that the minimum amount of taxes and fines is equal to $-C^*$, meaning that if the optimal policy calls for a transfer from the government to households, that must be at most C^* . This assumption simplifies the proof of the existence of optimal mechanisms and also is realistic. There is no reason for the government to redistribute more goods than the minimum to for an agent to reach the poverty line.

II. Optimal Tax-cum-Audit Policy

This section characterizes the optimal income tax, fines, auditing probabilities, consumption and leisure for both types of households, when poverty and tax evasion are integrated in a general income tax model. In an environment where enforcement is costly and incomes are observable only through an audit cost, the social planner maximizes

$$\begin{aligned}
 W &= EU_{pp} + \delta EU_{rr} & (1) \\
 &= (1 - p_p)u(Y_p - T_p) + p_p u(Y_p - F_p) + v_p(Y_p) \\
 &\quad + \delta [(1 - p_r)u(Y_r - T_r) + p_r u(Y_r - F_r) + v_r(Y_r)] \\
 &\quad - \beta [P(C_p^e, C_r^e)]
 \end{aligned}$$

where δ is the relative social weights imposed on the rich and $\beta = \beta_p + \delta\beta_r$ is the aggregate aversion to poverty. The planner's problem is also subject to the revenue and incentive compatibility constraints. Hence, the social planner maximizes equation (1) with respect to $Y_p, Y_r, p_p, p_r, T_p, T_r, F_p$ and F_r , subject to the following self-selection constraints

$$EU_{rr} \geq EU_{rp}, \quad (2)$$

$$EU_{rr} \geq V_{rr}, \quad (3)$$

$$EU_{rr} \geq V_{rp}, \quad (4)$$

$$EU_{pp} \geq V_{pp}. \quad (5)$$

and the revenue constraint

$$N_p[(1 - p_p)T_p + p_p F_p] + N_r[(1 - p_r)T_r + p_r F_r] - A(N_p p_p + N_r p_r) \geq \bar{R} \quad (6)$$

where \bar{R} stands for the necessary tax revenue, N_p and N_r are the proportion of poor and rich individuals in this economy, respectively. We ignore an ‘‘upward’’ incentive constraint, i.e., the constraint that the poor individual tries to mimic the rich is not binding. Note that the incen-

tive compatibility constraints force all consumers to correctly state income in equilibrium which characterizes the optimum tax-cum-audit policy.

The constraint (2) is the usual self-selection constraint when incomes are not observable and constraints (3), (4) and (5) are so-called the moral hazard conditions and have to be satisfied to avoid tax evasion. Self-selection constraints (2), (3) and (4) ensure that a rich/skilled household prefers a truthful statement of his type and income than mimicking the poor/unskilled person and his associate income, misreporting his income while declaring a rich/skilled person and misreporting his income and his type, respectively. The constraint (5) ensures that poor/unskilled individual prefers a truthful statement of his type and income than misreporting his income.

The Lagrangian expression for the planner's problem when enforcement is costly and incomes are observable only through an audit cost is written as

$$\begin{aligned}
\Lambda = & [(1 - p_p)u(Y_p - T_p) + p_p u(Y_p - F_p) + v_p(Y_p)] \\
& + \delta [(1 - p_r)u(Y_r - T_r) + p_r u(Y_r - F_r) + v_r(Y_r)] \\
& - \beta [P(Y_p - (1 - p_p)T_p - p_p F_p, Y_r - (1 - p_r)T_r - p_r F_r)] \\
& + \lambda_1 \left[\begin{array}{l} (1 - p_r)u(Y_r - T_r) + p_r u(Y_r - F_r) + v_r(Y_r) - \beta_r P^* \\ -(1 - p_p)u(Y_p - T_p) - p_p u(Y_p - F_p) - v_p(Y_p) + \beta_p \bar{P} \end{array} \right] \\
& + \lambda_2 [(1 - p_r)u(Y_r - T_r) + p_r u(Y_r - F_r) + v_r(Y_r) - \beta_r P^* - V_{rr}] \\
& + \lambda_3 [(1 - p_r)u(Y_r - T_r) + p_r u(Y_r - F_r) + v_r(Y_r) - \beta_r P^* - V_{rp}] \\
& + \lambda_4 [(1 - p_p)u(Y_p - T_p) + p_p u(Y_p - F_p) + v_p(Y_p) - \beta_p P^* - V_{pp}] \\
& + \mu [N_p[(1 - p_p)T_p + p_p F_p] + N_r[(1 - p_r)T_r + p_r F_r] - A(N_p p_p + N_r p_r) - \bar{R}]
\end{aligned}$$

where $P^* = P(Y_p - (1 - p_p)T_p - p_p F_p, Y_r - (1 - p_r)T_r - p_r F_r)$, $\bar{P} = P(C_p^e, C_p^e)$, λ_1 denotes the Lagrangian multiplier for the self-selection constraint, λ_2 , λ_3 and λ_4 represent the Lagrange multiplier of moral hazard constraints and μ is the marginal cost of an additional unit of revenue in utility terms.

To improve readability, the first order conditions of this problem with respect to Y_p , Y_r , T_p , T_r , F_p , F_r , p_p and p_r are presented in the Appendix.

II.1. Optimal Audits

We focus our analysis on the more interesting case of an interior solution, where it is possible to audit both types of agents. Although not considered, a corner solution of no audits in this environment is possible for instance, if audit costs are very high. In this case, the only feasible tax would be a uniform lump-sum tax without redistribution concerns.

Before presenting our main results, it is necessary and useful to determine what constraints are binding or not, which in turn depends on assumptions regarding the poverty measure. Let $Y(i, p_k, T_k, F_k)$ be the income that would be chosen optimally by type i in the case where he states to be of type k but never gets caught misreporting his income, even if he is audited. Of course, this is not a situation that can actually happen, as income misreporting is always discovered by an audit. The results are stated below:

Lemma 1 *(i) $\lambda_3 = \lambda_4 = 0$ is not possible and (ii) If P is an absolute measure, then $Y_r = Y(r, p_r, T_r, F_r)$ which implies $\lambda_2 = 0$.*

The intuition behind Lemma 1 is straightforward. Its first part says that at least one moral hazard constraint should be binding. If that was not the case one could always increase welfare by reducing the probability of auditing the poor, p_p . This role of auditing is to ensure compliance with these constraints. Turning to the second claim, in the case of an absolute poverty measure, the action of the richest individuals plays no role in poverty and therefore, imposing a distortion on them would only cause a welfare loss. Optimally, those individuals should receive their income that they would choose for themselves under the (p_r, T_r, F_r) scheme.

We are now in a position to address the optimal choice of the audit probabilities. In an economy with tax evasion and poverty, the government's aim is to prevent individuals from reporting a lower level of income than the one assigned for their skill-type. The optimal policy could entail positive auditing probabilities on both types when the poverty measure is relative. The intuition behind this result is the fact reported rich individual affect this measure of poverty and they might feel attracted to misreport their income, creating the necessity to audit the rich as well as the poor. Our main result is formally stated and proved in Proposition 1.

Proposition 1 For absolute poverty measures, it is optimal not to audit the rich, $p_r = 0$. For relative poverty measures, the optimal auditing implies that rich individuals could face stochastic audits probabilities, i.e., $1 > p_r \geq 0$. In particular, if $Y_r \neq Y(r, 0, T_r, F_r)$, then $p_r > 0$. The poor individuals should face an optimal auditing probability less than one, $p_p < 1$.

Proof. We first prove the optimal auditing policy for rich individuals. The first order conditions of the planner's problem with respect to p_r is

$$\left[\begin{array}{l} (\delta + \lambda_1 + \lambda_2 + \lambda_3) [u_r^F - u_r^T] + (\beta + \beta_r \lambda_1 + \beta_r \lambda_2 + \beta_r \lambda_3 + \beta_p \lambda_4) (F_r - T_r) \frac{dP^*}{dC_r} \\ + \mu N_r (F_r - T_r - A') + \lambda_2 u_{rr}^T - \lambda_2 \beta_r (Y_r - T_r) \frac{dP_{rr}}{dC_r} - \lambda_4 (F_r - T_r) \beta_p \frac{dP_{pp}}{dC_r} \end{array} \right] = 0 \quad (7)$$

1) *Absolute poverty measure.* In this case, it is clear that $\frac{dP}{dC_r} = 0$ and $\lambda_2 = 0$. The later result implies that $F_r = T_r$ (see Proposition 3). Therefore, we have $\partial\Lambda/\partial p_r = -\mu N_r A' < 0$. Therefore it is optimal to set p_r at its minimum of 0.

2) *Relative poverty measure.* We first show that $p_r < 1$. Suppose that $p_r = 1$. Then, constraint (3) is automatically satisfied and $\lambda_2 = 0$, which implies $F_r = T_r$. Equation(7) becomes $-\mu N_r A' < 0$. So it is optimal to reduce p_r . Now, suppose that $p_r = 0$ and consider constraint (3). We need to verify if $EU_{rr} \geq V_{rr}$. Since $p_r = 0$, it becomes

$$u(Y_r - T_r) + v_r(Y_r) - \beta_r P(C_p, C_r) \geq u(Y - T_r) + v_r(Y) - \beta_r P(C_p, C'_r)$$

with $Y = \tilde{Y}(r, 0, T_r) = Y(r, 0, T_r, F_r)$ and $C'_r = Y - T_r$. Using the definition of $\tilde{Y}(r, 0, T_r)$,

$$u(Y_r - T_r) + v_r(Y_r) - \beta_r P(C_p, C_r) < u(Y - T_r) + v_r(Y) - \beta_r P(C_p, C'_r)$$

if $Y_r \neq Y(r, 0, T_r, F_r)$. Therefore, $p_r > 0$ as long as $Y_r \neq Y(r, 0, T_r, F_r)$.⁷

We not turn to the optimum audit probability for poor individuals. Suppose that $p_p = 1$. Then constraints (4) and (5) are automatically satisfied, i.e., an individual that declares to be poor cannot cheat since s/he will be audited for sure. This implies that $\lambda_3 = \lambda_4 = 0$, a contradiction of our Lemma 1. ■

An optimal positive auditing probability on the skilled individuals is quite surprising but intuitive. With a relative poverty measure, skilled individuals have an incentive to misreport their income (λ_2 can be different from zero in this set up). This occurs because in our economy, with tax evasion and poverty, an increase in the consumption of the rich may lead to social welfare losses. This implies that a rich individual could attempt to misreport his income even if he does not misdeclare his type in order to have a larger consumption level than the one assigned by the planner. Therefore it becomes necessary to audit incomes on those that declare to be skilled.

On the other hand, the intuition for having auditing probabilities less than one in both individuals is similar to the one provided in Cremer and Gahvari (1996). Suppose $p_k = 1$, then individuals will have to pay F_k for certain. On the other hand they can have the same certain net income with random audits provided they pay the same amount of taxes. This policy is welfare improving because it decreases audit costs and is feasible because cheating by not choosing the assigned income is a strictly dominated strategy for a specific audit probability (Mookherjee and Png, 1989).

II.2. Optimal Taxes

In order to characterize the optimal taxes for both individuals we have to first define the Marginal Tax Rate (MTR) associated to our problem. Let the MTR for an agent of type k be defined as $MTR_k = 1 + \left[\left(\frac{\partial v_k(Y)}{\partial Y} \right) / \left(E \frac{\partial u_k}{\partial C} - \beta_k \frac{dP^*}{dC_k} \right) \right]$ derived from the expression $E \frac{\partial u_k}{\partial C} + \frac{\partial v_k(Y)}{\partial Y} - \beta_k \frac{dP^*}{dC_k} = 0$, which simply represents the optimality condition when an individual of type k chooses his income without any policy intervention. Using the first order conditions of the planner's problem with respect to Y_r and Y_p (equations (11) and (10), respectively, in Appendix A.1) and after some simplifications we obtain the following marginal tax rates on rich and poor individuals,

respectively:

$$MTR_r = \frac{\beta_p \frac{dP^*}{dC_r} + \lambda_4 \beta_p \left(\frac{dP^*}{dC_r} - \frac{dP_{pp}}{dC_r} \right)}{(\delta + \lambda_1 + \lambda_2 + \lambda_3) \left(E \frac{\partial u_r}{\partial C} - \beta_r \frac{dP^*}{dC_r} \right)} \quad (8)$$

$$MTR_p = \frac{\left(\lambda_1 \left[E \frac{\partial u_p}{\partial C} + \frac{\partial v_r(Y_p)}{\partial Y} \right] + \delta \beta_r \frac{dP^*}{dC_p} + \beta_r \lambda_1 \left(\frac{dP^*}{dC_p} - \frac{d\bar{P}}{dC_p} \right) + \beta_r \lambda_2 \left(\frac{dP^*}{dC_p} - \frac{dP_{rr}}{dC_p} \right) + \beta_r \lambda_3 \left(\frac{dP^*}{dC_p} - \frac{dP_{rp}}{dC_p} \right) \right)}{(1 + \lambda_4) \left(E \frac{\partial u_p}{\partial C} - \beta_p \frac{dP^*}{dC_p} \right)} \quad (9)$$

In our economy, for an absolute measure of poverty we can easily observe that the labor supply decision of the rich should not be distorted. On the other hand, if a relative poverty measure is considered, it is possible to characterize conditions under which both agents, rich and poor, face either positive, negative or zero marginal tax rates. The following proposition summarizes these findings.

Proposition 2 *i) If P is an absolute measure:*

a) $MTR_r = 0$;

b) $sign(MTR_p) = sign \left(\lambda_1 \left[E \frac{\partial u_p}{\partial C} + \frac{\partial v_r(Y_p)}{\partial Y} \right] + \delta \beta_r \frac{dP^*}{dC_p} \right)$.

ii) If P is a relative measure:

a) $sign(MTR_r) = sign \left(\beta_p \frac{dP^*}{dC_r} + \lambda_4 \beta_p \left(\frac{dP^*}{dC_r} - \frac{dP_{pp}}{dC_r} \right) \right)$;

b) $sign(MTR_p) = sign \left(\lambda_1 \left[E \frac{\partial u_p}{\partial C} + \frac{\partial v_r(Y_p)}{\partial Y} \right] + \delta \beta_r \frac{dP^*}{dC_p} + \beta_r \lambda_1 \left(\frac{dP^*}{dC_p} - \frac{dP^{(C_p^e)}}{dC_p} \right) + \beta_r \lambda_2 \left(\frac{dP^*}{dC_p} - \frac{dP_{rr}}{dC_p} \right) + \beta_r \lambda_3 \left(\frac{dP^*}{dC_p} - \frac{dP_{rp}}{dC_p} \right) \right)$

Proof. The first claim, associate with absolute poverty measure, can be easily depicted if we impose $(dP(\cdot)/dC_r) = 0$ on equations (8) and (9). The numerator of equation (8) becomes null. Equation (9) becomes $\lambda_1 [E \partial u_p / \partial C + \partial v_r(Y_p) / \partial Y] + \delta \beta_r dP^* / dC_p / [(1 + \lambda_4)(\partial u_p / \partial C) - \beta_p dP^* / dC_p]$. Since the denominator is positive, the $sign(MTR_p)$ is determined by the numerator. Turning to second claim, for relative poverty measure it is sufficient to determine the sign of the numerator of expressions (8) and (9), once the denominators are positive. ■

This proposition deserves special attention as our results contrasts sharply with the ones in the existing optimum tax literature. We consider the MTR for each type separately. For the

rich, we have the surprising result that for a relative poverty measure MTR_r can be negative, positive or zero. The non-intuitive case of MTR_r negative is the less likely: it would occur only if it is very costly to satisfy constraint (5) (high λ_4) and an additional dollar for the rich has much more effect on relative poverty measure when the poor is misreporting his income than when he is not ($dP_{pp}/dC_r > dP^*/dC_r$). More precisely, we have a negative MTR_r when $dP_{pp}/dC_r > (dP^*/dC_r) * (1 + \lambda_4)/\lambda_4$. In that case we should subsidize the rich in a effort to make it less appealing for the poor to misreport his income. On the other hand if these effects are identical $dP_{pp}/dC_r > (dP^*/dC_r) * (1 + \lambda_4)/\lambda_4$ or with an absolute poverty measure (in which case $dP^*/dC_r = dP_{pp}/dC_r = 0$), we reproduce the standard result in the literature (Stiglitz, 1982, Seade, 1977 and Cremer and Gahvari, 1996), which is no distortion at the top. Since the rich individuals do not affect the absolute poverty measure, no distortion is necessary on their labor supply decision. Lastly, the most common case would be where the impact of the rich on the poverty measure is similar, regardless of the action of the poor ($dP^*/dC_r - dP_{pp}/dC_r$ close to zero). Then, the dominating effect is $\beta_p dP^*/dC_r$, which is positive. We therefore impose a positive MTR, as a distortion should be imposed on his/her labor supply decision in order to have more resources at the margin to redistribute. This last result seems more likely since it allows to distort the rich behavior in order to reduce relative poverty measure in a direct manner. Still, it is interesting to see that there exists a theoretical possibility to subsidize the rich, even though the relative poverty is a public bad.

Regarding the poor, it is important to mention that the component $\delta\beta_r dP^*/dC_p$ is the only one for which the sign is unambiguous, i.e., negative. This term is also the most relevant since it captures the net effect of poor consumption on welfare grounds. The other effects come through the constraints. In the case of relative poverty measure, the marginal tax on the poor would be positive ($MTR_p > 0$) if by increasing Y_p it would make very attractive for the rich

(i) to declare he is poor and behave like one ($\lambda_1[E\partial u_p/\partial C + \partial v_r(Y_p)/\partial Y] + \beta_r\lambda_1[dP^*/dC_p - dP_{C_p}/dC_p]$),

(ii) to declare he is rich and misreport his/her income ($\beta_r\lambda_2[dP^*/dC_p - dP_{rr}/dC_p]$ and

(iii) to declare he is poor and misreport his/her income $(\beta_r \lambda_3 (dP^*/dC_p - dP_{rp}/dC_p))$.

Nevertheless all the terms in brackets are undetermined and a sufficient condition for $MTR_p > 0$ is that the effect of the poor's consumption on the poverty measure when he does not misreport his income is larger than its correspondent effect (i) on the poverty measure when he/she misreport his/her income, (ii) when the rich ones misreport their income, (iii) when the rich misreport their income and their type, and the direct effect of consumption of poor individuals on their utility is larger than its counterpart on labor disutility for the rich. Finally the sum of all these effects should be greater than $\delta\beta_r dP^*/dC_p$.

The fact that a zero marginal tax rate on the poor could be optimal implies that the sum of the numerator components should be zero. A straightforward example would be when no constraints on the rich are binding ($\lambda_1 = \lambda_2 = \lambda_3 = 0$) and there is no poverty concern on the part of rich individuals.

More reasonable assumptions lead to the conclusion that $MTR_p < 0$. For example, this is the case if we assume that the effect of the poor's consumption on the poverty measure when poor individuals are telling the truth (dP^*/dC_p) is larger than its counterpart effect on poverty when

(i) poor agents are misreporting income (dP_{C_p}/dC_p),

(ii) the rich ones misreport their income (dP_{rr}/dC_p) and

(iii) rich individuals misreport their type and their income (dP_{rp}/dC_p).

II.3. Optimal Fines

This section presents the optimal fines in our set up. Our Lemma 1 already addresses that $F_p \neq T_p$ may not happen. In the presence of poverty and tax evasion, an interesting result emerges, namely, rewarding the truth telling poor is not necessarily optimal. Our optimum fine policy allows for punishing agents declared to be poor and do not misreport income. We also find that fines imposed on the rich should not exceed the correspondent taxes. Our last proposition states and proves this finding.

Proposition 3 *The optimum fines imposed on poor individuals can be greater or lower than the correspondent taxes, but never equal. For the rich individuals, those fines should be either lower or equal to the taxes, but never greater.*

Proof. We start proving the first claim. Compare the first order conditions of the planner's problem with respect to T_p and T_r , respectively:

$$(1 - p_p) \left[\begin{array}{c} -(1 + \lambda_4 - \lambda_1) \frac{\partial u_p^T}{\partial C} + (\beta + \beta_r \lambda_1 + \beta_r \lambda_2 + \beta_r \lambda_3 + \beta_p \lambda_4) \frac{dP^*}{dC_p} \\ + \mu N_p + \lambda_1 \beta_r \frac{dP_{C_p}}{dC_p} - \lambda_2 \beta_r \frac{dP_{rr}}{dC_p} - \lambda_3 \beta_r \frac{dP_{rp}}{dC_p} + \lambda_3 \frac{\partial u_r^{T^*}}{\partial C} - \lambda_3 \beta_r \frac{dP_{rp}}{dC_r} + \lambda_4 \frac{\partial u_p^{T^*}}{\partial C} - \lambda_4 \beta_p \frac{dP_{pp}}{dC_p} \end{array} \right] = 0$$

$$p_p \left[\begin{array}{c} -(1 + \lambda_4 - \lambda_1) \frac{\partial u_p^F}{\partial C} + (\beta + \beta_r \lambda_1 + \beta_r \lambda_2 + \beta_r \lambda_3 + \beta_p \lambda_4) \frac{dP^*}{dC_p} \\ + \mu N_p + \lambda_1 \beta_r \frac{dP_{poor}}{dC_p} - \lambda_2 \beta_r \frac{dP_{rr}}{dC_p} - \lambda_3 \beta_r \frac{dP_{rp}}{dC_p} \end{array} \right] = 0$$

Rearrange the above equations as follows:

$$\frac{\partial u_p^T}{\partial C} = \frac{Z + \lambda_3 \frac{\partial u_r^{T^*}}{\partial C} - \lambda_3 \beta_r \frac{dP_{rp}}{dC_r} + \lambda_4 \frac{\partial u_p^{T^*}}{\partial C} - \lambda_4 \beta_p \frac{dP_{pp}}{dC_p}}{(1 + \lambda_4 - \lambda_1)}$$

$$\frac{\partial u_p^F}{\partial C} = \frac{Z}{(1 + \lambda_4 - \lambda_1)}$$

where $Z = (\beta + \beta_r \lambda_1 + \beta_r \lambda_2 + \beta_r \lambda_3 + \beta_p \lambda_4) dP^*/dC_p + \mu N_p + \lambda_1 \beta_r d\bar{P}/dC_p - \lambda_2 \beta_r dP_{rr}/dC_p - \lambda_3 \beta_r dP_{rp}/dC_p$. It is clear that $\partial u_r^{T^*}/\partial C - \beta_r dP_{rp}/dC_r > 0$ and $\partial u_p^{T^*}/\partial C > 0$, $-dP_{pp}/dC_p > 0$. Since we cannot have $\lambda_3 = \lambda_4 = 0$, then $\lambda_3 \partial u_r^{T^*}/\partial C - \lambda_3 \beta_r dP_{rp}/dC_r + \lambda_4 \partial u_p^{T^*}/\partial C - \lambda_4 \beta_p dP_{pp}/dC_p > 0$. This directly implies that $F_p \neq T_p$. If $1 + \lambda_4 > \lambda_1$, then $\partial u_p^T/\partial C > \partial u_p^F/\partial C$, which immediately implies that $F_p < T_p$. On the other hand if $1 + \lambda_4 < \lambda_1$, then $\partial u_p^T/\partial C < \partial u_p^F/\partial C$, which immediately implies that $F_p > T_p$.

Turning to the second claim, we have that if $\lambda_2 = 0$, then $F_r = T_r$. To prove that $F_r < T_r$, we have to compare the first order conditions of the planner's problem with respect to T_r and F_r (equations (13), (15) in Appendix A.1). The only differences between them are the terms $\lambda_2 \partial u_{rr}^T/\partial C - \lambda_2 \beta_r dP_{rr}/dC_r$. If $\lambda_2 = 0$, then these equations are identical which implies $F_r = T_r$. On the other hand, if $\lambda_2 > 0$, then $\lambda_2 [\partial u_{rr}^T/\partial C - \beta_r dP_{rr}/dC_r] > 0$, where $\partial u_r^T/\partial C_r > \partial u_r^F/\partial C_r$ which implies that $F_r < T_r$. ■

The first part of this proposition essentially says that if the cost of satisfying constraint (2) is large compared to the cost of satisfying constraint (5), i.e., $\lambda_1 > 1 + \lambda_4$, then we would have the unexpected result that $F_p > T_p$. This would happen because skilled individuals have a large incentive to mimic the unskilled (high λ_1) and therefore a punishment on those that claimed and are observed to be poor is necessary, given that the government cannot identify their types only incomes. This situation would make less attractive for the rich to misreport his type. Alternatively, if poor individuals have a large incentive to misreport their income ($\lambda_4 > \lambda_1 - 1$), then it is optimal to reward those that do not misreport income, i.e., $F_p < T_p$. Concerning the rich, we only should reward them when they have incentive to misreport their type, otherwise the fines should be the same as taxes imposed on them.

III. Conclusion

This paper integrates poverty consideration and tax evasion into an optimum general income tax problem with endogenous labor supply. The notion that both rich and poor individuals can evade taxes and policymakers can adopt different measures of poverty have important implications for the question of optimal income tax design. We show that the optimum tax-cum audit policy depend on whether poverty is measured in absolute or in relative terms. For the first measure, we replicate most of the results presented in Cremer and Gahvari (1995) and Wane (2001) such as 'no distortion and no auditing at the top', and an undetermined marginal tax rate with random auditing for the ones declared to be poor. With an absolute poverty measure, the income of those claimed to be rich does not affect (distort) the provision of the public bad, aggregate poverty. In this case the optimum income tax-audit policy does not result in any distortion on the rich. However, for the ones claiming to be poor, they could present positive, negative or zero marginal taxes depending on the extent that their consumption affect the poverty measure compared to the incentive that the rich might have to mimic them. Last, the poor individuals should be audited randomly.

Our main contribution is to show that three different results emerge when we consider a relative poverty measure. First, we find that the optimal marginal income tax rate on both rich and poor individuals are undetermined; they can be positive, negative or zero. Second, those that declare to

be rich can be audited randomly, similar to their counterpart poor ones. Lastly, honesty may be punished as well as rewarded. As the rich individuals can affect the poverty measure, one should impose a distortion on their decision to work so that it makes less attractive for them to increase the poverty distance. Nevertheless, if they are really tempted to mimic the poor, the optimum policy suggest that a subsidy on them could be in place to make it less attractive. This result although less obvious is still possible.

The main distinction of this paper to the existing literature is that rich individuals could be audited. That might happen because now rich individuals consumption affect the poverty measure and could have their labor supply decision distorted. In turn, they have incentive to misreport income which could be audited to enforce truth telling. Finally, if the rich have a strong incentive to mimic the poor, then we should impose a larger fine on those (poor) to discourage misbehavior of the rich, i.e., we should punish honest poor. But if the poor ones are more willing to misreport their income then we should reward them for being honest. Our findings shed a light on some potentially important policy issues. We have shown that the measurement of poverty is crucial for the optimal income taxation design and it is worthwhile to further investigate the reasons behind different choices of standard of living to measure poverty.

Acknowledgments

We would like to thank Firouz Gahvari, Peter Townley and Anne Villamil for their valuable comments and suggestions. We have also benefited from discussions with seminar participants at the Brazilian Economic Association, Midwest Economic Association Meetings and participants at the 66th Congress of International Tax and Public Finance. All remaining errors are ours.

Notes

¹In this paper, the terms informal activities and tax evasion refer to all income generating activities which do not comply with the tax obligations, tax evasion and non-compliance with economic legislation.

²Besley and Coate (1992, 1995) show that a wage subsidy can be an optimal policy only when abilities are perfectly observable. Leblanc (2004) concludes that an universal provision of training is better than a negative income tax. In these studies, the program is chosen if it has the minimum cost amongst all the alternatives.

³See, for instance, Ravallion (1994) for further discussion.

⁴Kanbur, Keen and Tuomala (1994) use the same formulation to define aggregate poverty. However they consider the case that minimizing poverty is the only objective of the government.

⁵Consider Wane (2001)'s poverty function but with a relative poverty line instead $(C - C^*(C_p, C_r))^2$. According to United Nations and World Bank standards, a person is poor when his or her income is less than 60 percent of the average per capita income in the country of residence. Assume then that $C^*(C_p, C_r) = (0.6)[(C_p + C_r)/2]$. In this case (and related ones), our assumption that the direct effect dominates the indirect effect is reasonable. There are important measurement issues in selecting the standard of living (see Citro and Michael, 1995; Foster, 1998; Muller, 2006).

⁶The magnitude of penalties is the object of debate in the enforcement literature. If the government is free to choose the penalties, Becker (1968), Chander and Wilde (1998), among others, have shown that (extremely) severe penalties are optimal. However, less-than-maximum fines can be optimal when enforcement is uncertain (Polinsky and Shavell, 2005) or social norms impose economic restrictions on the penalty function (Marhuenda and Ortuno-Ortin, 1997). See Chander (2007) for a discussion on income tax evasion, penalty and risk aversion.

⁷Note that $Y_r = Y(r, 0, T_r, F_r)$ is possible if $(\gamma \frac{dP^*}{dC_r} - \lambda_4 \beta_p \frac{dP_{pp}}{dC_r}) / (\delta + \lambda_1 + \lambda_2 + \lambda_3) = \beta_r \frac{dP^*}{dC_r}$. We argue that this would happen only by coincidence.

References

- [1] Becker, G., 1968. Crime and Punishment: an economic approach. *Journal of Political Economy* 76 (2), 169-217.
- [2] Besley, T. and S. Coate (1992). Workfare versus Welfare: Incentive Arguments for Work Requirements in Poverty-Alleviation Programs, *American Economic Review* 82, 249–261.

- [3] Besley, T. and S. Coate (1995). The design of income maintenance programs, *Review of Economic Studies* 62, 187-221.
- [4] Border, K. and J. Sobel (1987). Samurai accountant: a theory of auditing and plunder, *Reviews of Economic Studies* 54, 525-540.
- [5] Chander, P., L. Wilde (1998). A General Characterization of Optimal Income Tax. *Review of Economics Studies* 65(1), 165-183.
- [6] Chander, P. (2007). Income Tax Evasion and the Fear of Ruin. *Economica* 74, 315–328.
- [7] Cremer, H. and Gahvari, F. (1994). Tax evasion, concealment and the optimal linear income tax, *Scandinavian Journal of Economics* 96, 219-239.
- [8] Cremer, H. and Gahvari, F. (1996). Tax evasion and the optimum general income tax, *Journal of Public Economics* 60, 235-249.
- [9] Foster, J. (1998). absolute versus Relative Poverty” *American Economic Review, Papers and Proceedings* 88, 335-341.
- [10] Garfinkel, I. (1973). A skeptical note on “The optimality” of wage subsidy programs”, *American Economic Review* 63, 447-453.
- [11] International Labour Office (2010). *Global employment trends, January 2010* (Geneva)
- [12] Kanbur, R., M. Keen and M. Tuomala (1994). Optimal non-linear income taxation for the alleviation of income poverty, *European Economic Review* 38, 1613-1632.
- [13] Kaplow, Louis (1990). Optimal taxation with costly enforcement and evasion, *Journal of Public Economics* 43, 221-236.
- [14] Kesselman, J. (1971). Conditional Subsidies in Income Maintenance, *Western Economic Journal*, 9,1-20.
- [15] Leblanc, G. (2004). Optimal income maintenance and the “unemployable”, *Journal of Public Economic Theory* 6(3), 509-535.
- [16] Maloney, W. (2004). Informality Revisited. *World Development*, 32(7), 1159-1178.
- [17] Marhuenda, F., I. Ortuno-Ortin, 1997. Tax enforcement problems. *Scandinavian Journal of Economics* 9(91), 61-72.
- [18] Mirrlees, J. (1971). An elaboration in the theory of optimal income taxation. *Review of Economic Studies* 28, 175-208.
- [19] Moffit, R. (2003). The negative income tax and the evolution of U.S. welfare Policy, NBER working paper 9751.
- [20] Mookherjee D. and I. Png (1989). Optimal auditing, insurance, and redistribution, *Quarterly Journal of Economics* 103, 399-415.
- [21] Muller, C. (2006). Defining Poverty Lines as a Fraction of Central Tendency, *Southern Economic Journal* 72(3), 720-729.

- [22] Nichols, A. L. and R. Zeckhauser (1982). Targeting Transfer through Restrictions on Recipient, *American Economic Review* 78, 372–377.
- [23] Pirttila, J. and M. Tuomala (2004). Poverty alleviation and tax policy. *European Economic Review* 48, 1075-1090.
- [24] Polinsky, M., S. Shavell, 2000. The economic theory of public enforcement of law. *Journal of Economic Literature*, 38(1), 45-76.
- [25] Ravallion, M. (1994). Poverty Comparisons, in Harwood Academic Press *Fundamentals of Pure and Applied Economics*, 56.
- [26] Sandmo, A. (1981). Income tax evasion, labour supply, and the equity-efficiency tradeoff, *Journal of Public Economics* 16, 265-288.
- [27] Sandmo, A. (2005). The theory of tax evasion: a retrospective view, *National Tax Journal* 58(4), 643–663
- [28] Schneider, F. and Enste, H. (2000). Shadow Economies: Size, causes and consequences, *Journal of Economic Literature* 38, 77-114.
- [29] Simler, K. and C. Arndt (2007). Poverty comparisons with absolute Poverty Lines Estimated from Survey Data, *Review of Income and Wealth* 53(2), 275-294.
- [30] Wane, W. (2001). Optimal general income tax and poverty as a public bad, *Journal of Public Economics* 82(2), 271-299.
- [31] Zeckhauser, R. (1971). Optimal mechanisms for income transfer, *American Economic Review* 61, 324-334.

Appendix

A.1 - Planner's problem: first-order conditions

To improve readability, the first order conditions of this problem with respect to $Y_p, Y_r, T_p, T_r, F_p, F_r, p_p$ and p_r are, respectively

$$\begin{bmatrix} (1 + \lambda_4) \left[E \frac{\partial u_p}{\partial C} + \frac{\partial v_p(Y_p)}{\partial Y} \right] - \lambda_1 \left[E \frac{\partial u_p}{\partial C} + \frac{\partial v_r(Y_r)}{\partial Y} \right] \\ -\gamma \frac{dP^*}{dC_p} + \lambda_1 \frac{d\bar{P}}{dC_p} - \lambda_2 \frac{\partial V_{rr}}{\partial Y_p} - \lambda_3 \frac{\partial V_{rp}}{\partial Y_p} - \lambda_4 \frac{\partial V_{pp}}{\partial Y_p} \end{bmatrix} = 0, \quad (10)$$

$$\begin{bmatrix} (\delta + \lambda_1 + \lambda_2 + \lambda_3) \left[E \frac{\partial u_r}{\partial C} + \frac{\partial v_r(Y_r)}{\partial Y} \right] - \gamma \frac{dP^*}{dC_r} \\ -\lambda_2 \frac{\partial V_{rr}}{\partial Y_r} - \lambda_3 \frac{\partial V_{rp}}{\partial Y_r} - \lambda_4 \frac{\partial V_{pp}}{\partial Y_r} \end{bmatrix} = 0, \quad (11)$$

$$\begin{bmatrix} -(1 + \lambda_4 - \lambda_1)(1 - p_p) \frac{\partial u_p^T}{\partial C} + \gamma(1 - p_p) \frac{dP^*}{dC_p} \\ + \mu N_p(1 - p_p) - \beta_r \lambda_1(1 - p_p) \frac{d\bar{P}}{dC_p} - \lambda_2 \frac{\partial V_{rp}}{\partial T_p} - \lambda_3 \frac{\partial V_{rp}}{\partial T_p} - \lambda_4 \frac{\partial V_{pp}}{\partial T_p} \end{bmatrix} = 0, \quad (12)$$

$$\begin{bmatrix} -(\delta + \lambda_1 + \lambda_2 + \lambda_3)(1 - p_r) \frac{\partial u_r^T}{\partial C_r} + \gamma(1 - p_r) \frac{dP^*}{dC_r} \\ + \mu N_r(1 - p_r) - \lambda_2 \frac{\partial V_{rr}}{\partial T_r} - \lambda_3 \frac{\partial V_{rp}}{\partial T_r} - \lambda_4 \frac{\partial V_{pp}}{\partial T_r} \end{bmatrix} = 0, \quad (13)$$

$$\begin{bmatrix} -(1 + \lambda_4 - \lambda_1)p_p \frac{\partial u_p^F}{\partial C} + \gamma p_p \frac{dP^*}{dC_p} \\ + \mu N_p p_p - \beta_r \lambda_1 p_p \frac{d\bar{P}}{dC_p} - \lambda_2 \frac{\partial V_{rr}}{\partial F_p} - \lambda_3 \frac{\partial V_{rp}}{\partial F_p} - \lambda_4 \frac{\partial V_{pp}}{\partial F_p} \end{bmatrix} = 0, \quad (14)$$

$$\begin{bmatrix} -(\delta + \lambda_1 + \lambda_2 + \lambda_3)p_r \frac{\partial u_r^F}{\partial C_r} + \gamma p_r \frac{dP^*}{dC_r} \\ + \mu N_r p_r - \lambda_2 \frac{\partial V_{rr}}{\partial F_r} - \lambda_3 \frac{\partial V_{rp}}{\partial F_r} - \lambda_4 \frac{\partial V_{pp}}{\partial F_r} \end{bmatrix} = 0 \quad (15)$$

$$\begin{bmatrix} (1 + \lambda_4 - \lambda_1) [u_p^F - u_p^T] + \gamma (F_p - T_p) \frac{dP^*}{dC_p} \\ + \mu N_p (F_p - T_p - A') - \beta_r \lambda_1 (F_p - T_p) \frac{d\bar{P}}{dC_p} - \lambda_2 \frac{\partial V_{rr}}{\partial p_p} - \lambda_3 \frac{\partial V_{rp}}{\partial p_p} - \lambda_4 \frac{\partial V_{pp}}{\partial p_p} \end{bmatrix} = 0, \quad (16)$$

$$\begin{bmatrix} (\delta + \lambda_1 + \lambda_2 + \lambda_3) [u_r^F - u_r^T] + \gamma (F_r - T_r) \frac{dP^*}{dC_r} \\ + \mu N_r (F_r - T_r - A') - \lambda_2 \frac{\partial V_{rr}}{\partial p_r} - \lambda_3 \frac{\partial V_{rp}}{\partial p_r} - \lambda_4 \frac{\partial V_{pp}}{\partial p_r} \end{bmatrix} = 0. \quad (17)$$

where $u_k^T = u(Y_k - T_k)$, $u_k^F = u(Y_k - F_k)$ and $\gamma = \beta + \beta_r \lambda_1 + \beta_r \lambda_2 + \beta_r \lambda_3 + \beta_p \lambda_4$.

Note that $\partial V_{ik}/\partial x = \partial v_{ik}/\partial x$ where $v_{ik} = (1 - p_k)u(Y(i, p_k, T_k) - T_k) + v_i(Y(\cdot)) - \beta_i P_{ik}$, where $Y(i, p_k, T_k)$ denotes the maximum income an individual of type i pretending to be type k can obtain. Last denote $P_{rr} = P(Y_p - (1 - p_p)T_p - p_p F_p, (1 - p_r)(Y(r, p_r, T_r) - T_r))$, $P_{rp} = P(Y_p - (1 - p_p)T_p - p_p F_p, (1 - p_p)(Y(r, p_p, T_p) - T_p))$ and $P_{pp} = P((1 - p_p)(Y(p, p_p, T_p) - T_p), Y_r - (1 - p_r)T_r - p_r F_r)$.

Effects on the various V_{ik} are given in Table 1.

x	$\frac{\partial V_{rx}}{\partial w}$	$\frac{\partial V_{rp}}{\partial w}$	$\frac{\partial V_{pp}}{\partial w}$
Y_p	$-\beta_r \frac{dP_{rx}}{dC_p} > 0$	$-\beta_r \frac{dP_{rp}}{dC_p} > 0$	0
Y_r	0	0	$-\beta_p \frac{dP_{pp}}{dC_r} \leq 0$
T_p	$(1 - p_p) \beta_r \frac{dP_{rx}}{dC_p} \leq 0$	$-(1 - p_p) \left(\frac{\partial u_{rp}^T}{\partial C} - \beta_r \frac{dP_{rp}}{dC_p} - \beta_r \frac{dP_{rp}}{dC_r} \right)$	$-(1 - p_p) \left(\frac{\partial u_{pp}^T}{\partial C} - \beta_p \frac{dP_{pp}}{dC_p} \right) \leq 0$
T_r	$-(1 - p_r) \left(\frac{\partial u_{rx}^T}{\partial C} - \beta_r \frac{dP_{rx}}{dC_r} \right) < 0$	0	$\beta_p (1 - p_r) \frac{dP_{pp}}{dC_r} \geq 0$
F_p	$p_p \beta_r \frac{dP_{rx}}{dC_p} \leq 0$	$\beta_r p_p \frac{dP_{rp}}{dC_p} \leq 0$	0
F_r	0	0	$\beta_p p_r \frac{dP_{pp}}{dC_r} \geq 0$
p_p	$(F_p - T_p) \beta_r \frac{dP_{rx}}{dC_p}$	$-u_{rp}^T + \beta_r \left[(F_p - T_p) \frac{dP_{rp}}{dC_p} + (Y - T_p) \frac{dP_{rp}}{dC_r} \right]$	$-u_{pp}^T + \beta_p (Y - T_p) \frac{dP_{pp}}{dC_p} \leq 0$
p_r	$-u_{rr}^T + \beta_r (Y_r - T_r) \frac{dP_{rx}}{dC_r}$	0	$(F_r - T_r) \beta_p \frac{dP_{pp}}{dC_r}$

where $u_{ik}^T = u(Y - T_k)$, with Y chosen optimally by type i pretending to be type k .

A.2 - Proof of Lemma 1

Proof. We prove each claim separately.

(i) $\lambda_3 = \lambda_4 = 0$ is not possible.

Suppose that $\lambda_3 = \lambda_4 = 0$. We show that the consequences (1 – 4) of this assumption leads to a contradiction.

(1) $T_p = F_p$. Compare the equations (12) and (14). If $\lambda_3 = \lambda_4 = 0$, then they imply that $\frac{\partial u_p^T}{\partial C} = \frac{\partial u_p^F}{\partial C}$. By the properties of u , this is only possible if $T_p = F_p$. This leads to our second step:

(2) $p_p = 0$. Consider equation (16). From (1), we know that $T_p = F_p$. Then, $\frac{\partial \Lambda}{\partial p_p} = -\mu N_p A' < 0$. Therefore we set p_p to its minimum. That brings another consequence:

(3) $\lambda_1 = 0$. Given that $p_p = 0$, equation (4) becomes

$$(1 - p_r)u(Y_r - T_r) + p_r u(Y_r - F_r) + v_r(Y_r) - \beta_r P(Y_p - T_p, Y_r - (1 - p_r)T_r - p_r F_r) \geq u(\tilde{Y}(r, 0, T_p) - T_p) + v_r(\tilde{Y}(r, 0, T_p)) - \beta_r P(Y_p - T_p, \tilde{Y}(r, 0, T_p) - T_p)$$

while equation (2) becomes

$$(1 - p_r)u(Y_r - T_r) + p_r u(Y_r - F_r) + v_r(Y_r) - \beta_r P(Y_p - T_p, Y_r - (1 - p_r)T_r - p_r F_r) \geq u(Y_p - T_p) + v_r(Y_p) - \beta_r P(Y_p - T_p, Y_p - T_p)$$

Notice that when type r chooses $\tilde{Y}(r, 0, T_p)$, he can choose any income, including Y_p . Therefore, if constraint (4) is satisfied, so is constraint (2) and $\lambda_1 = 0$. Then we have another consequence:

(4) $T_p = Y_p$. Consider equation (10), we have $\frac{\partial u_p^T}{\partial C} + \frac{\partial v_p}{\partial Y} = (\beta + \lambda_2 \beta_r) \frac{dP^*}{dC_p} - \lambda_2 \beta_r \frac{dP_{rr}}{dC_p}$. Now consider, equation (12), which given the results above, simplifies to

$$\begin{aligned} \frac{\partial \Lambda}{\partial T_p} &= -\frac{\partial u_p^T}{\partial C} + (\beta + \lambda_2 \beta_r) \frac{dP^*}{dC_p} - \lambda_2 \beta_r \frac{dP_{rr}}{dC_p} + \mu N_p \\ &= \mu N_p - \frac{\partial v_p}{\partial Y} > 0 \end{aligned}$$

Therefore, we want to set T_p as high as possible, which is $T_p = Y_p$. Hence, $C_p = 0$, however, $\frac{\partial u(0)}{\partial C} \rightarrow \infty$, i.e., the benefit of increasing the consumption of the poor is infinity and the social weight attributed to the skilled individuals is less than infinity. Thus, it cannot be optimal for the planner to choose $C_p = 0$, a contradiction.

(ii) If P is an absolute measure, then $Y_r = Y(r, p_r, T_r, F_r)$ which implies $\lambda_2 = 0$.

Our second claim can be proven as follows. First we prove that If P is an absolute measure, then $Y_r = Y(r, p_r, T_r, F_r)$. Consider the equation (11)⁸

$$(\delta + \lambda_1 + \lambda_2 + \lambda_3) \left[E \frac{\partial u_r}{\partial C} + \frac{\partial v_r(Y_r)}{\partial Y} \right] - (\beta + \beta_r \lambda_1 + \beta_r \lambda_2 + \beta_r \lambda_3 + \beta_p \lambda_4) \frac{dP^*}{dC_r} + \lambda_4 \beta_p \frac{dP_{pp}}{dC_r} = 0$$

If P is an absolute measure, then $\frac{dP}{dC_r} = 0$ and the first-order condition becomes $\left[E \frac{\partial u_r}{\partial C} + \frac{\partial v_r(Y_r)}{\partial Y} \right] = 0$. This is the same condition as a rich agent optimally choosing his income when he faces a probability of being audited as p_r and taxes and fines defined as T_r and F_r and he never gets caught. Therefore, $Y_r = Y(r, p_r, T_r, F_r)$.

Next we show that this implies $\lambda_2 = 0$. Suppose $T_r = F_r$. Since P is an absolute measure,

$Y_r = Y(r, p_r, T_r, F_r)$. Notice that for $T_r = F_r$, we have $Y(r, p_r, T_r, F_r) = \tilde{Y}(r, 0, T_r)$ as in both cases the agent pays T_r in taxes for sure and never gets caught. Then, we have

$$\begin{aligned} EU_{kk} &= u(Y_r - T_r) + v_r(Y_r) - \beta_r P(Y_p - T_p, Y_r - T_r) \\ V_{rr} &= u(\tilde{Y}(r, 0, T_r) - T_r) + v_r(\tilde{Y}(r, 0, T_r)) - \beta_r P(Y_p - T_p, \tilde{Y}(r, 0, T_r) - T_r) \end{aligned}$$

Since $T_r \geq F_r$ (see Proposition 3), we have that $EU_{kk} \geq u(Y_r - T_r) + v_r(Y_r) - \beta_r P(Y_p - T_p, Y_r - T_r) = V_{rr}$, and income misreporting is never optimal. ■