Understanding “Instability” in Aggregate Money Demand: An Approach based on a Life-Cycle Model of Household Portfolio Choice

(Preliminary and Incomplete)

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Abstract

I study a model of portfolio choice over the life-cycle incorporating a transactions need of using monetary assets (MA), fixed costs of participation in non-monetary assets (NMFA) markets, and a realistic fluctuation of the return spread (the “opportunity cost” of holding MA). I estimate model parameters to match age profiles of portfolio positions and market participation in a dataset consisting of cross-sectional observations in 1962, 1983, and 1989. Simulated behavior closely resembles that in the cross-sections, and aggregation shows similarity to the time-series of the aggregate M2M (M2 minus small time deposits) in the data. My approach affords analysis of household and aggregate opportunity cost elasticities of MA at each point in time. While the elasticity of NMFA market participants is constant over time, that of non-participants varies moderately with the opportunity cost. Aggregate elasticity fluctuated dramatically between 1962 and 1989 following the movement of the opportunity cost. The contribution of the extensive margin of participation accounts for most of this fluctuation.

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1 Introduction

The traditional paradigm for modeling money demand is based on the trade-off of the transactions-value of money against the incremental yield available from holding non-monetary financial assets at the margin. This model, due to Baumol (1952) and Tobin (1956), has been invoked by a vast number of studies of the behavior of monetary aggregates. But looking at data at the level of households reveals that, while holding positive balances of monetary assets (MA), most hold no non-monetary financial assets (NMFA). This highlights two problems for the traditional empirical paradigm. First, the use of aggregate data becomes highly questionable when such extreme disparities of behavior exist at the micro-level. Second, as Mulligan and Sala-i-Martin (2002) state the point, “the relevant decision for the majority of U.S. households is not the fraction of assets to be held in interest-bearing form, but whether to hold any of such assets at all”. In particular, the existence of non-participants suggests that the Baumol-Tobin model is inapplicable at this level of observation; and, especially in light of the shortcomings of the empirical money demand literature, it calls into question whether it is applicable at a coarser level of aggregation.

Addressing the non-participation of households in important financial markets, financial economists often suggest the presence of fixed costs in explanation. Mulligan (1997) and Mulligan and Sala-i-Martin (2002, MS herafter) formulate a model of money demand incorporating per-period fixed costs of holding non-monetary financial assets. They show that this has important implications for our understanding of the interest elasticity of aggregates, which is closely related to the measurement of welfare costs accruing from inflation.

In this paper, I attempt to shed light on the nature of money demand by studying a standard life-cycle model with random income fluctuation and borrowing constraints augmented in two ways. First, I incorporate a “shopping technology”, as in McCallum and Goodfriend (1987), necessitating the use of MA in making transactions. Second, I allow for the possibility that payment of a fixed cost (of leisure time) may be necessary to access the NMFA market.

I estimate three parameterizations of the model by matching the moments of three cross-sections of data from the 1962 Survey of Financial Characteristics of Consumers and the 1983 and 1989 Surveys of Consumer Finances to the analogous moments from simulated data. The numerical implementation of the model is very rich, and incorporates realistic fluctuation of the spread of real rates of return on the two assets (the “opportunity cost”). Thus, households in the simulations respond to the same macroeconomic fluctuations faced by the households in the data. In
household and aggregate data, I compare MA in the model to the set of assets that comprise M2M (that is, M2 minus small time deposits).

I find that the patterns of NMFA market participation and non-participation cannot be satisfactorily explained without appeal to positive fixed costs. The cross-sectional data that I use does not offer strong evidence to discriminate between a model with a one time start-up cost, and a one with a per-period cost. Comparing more informally the aggregate series induced by each model, however, shows that a model with one-time “start-up” costs better captures a “shift” that is apparent in the data.

Conditional on participation, households’ MA balances under the model exhibit a constant elasticity, implying that their behavior may be well-described by the conventional inventory theoretic analysis. The MA balances of households that do not participate in the NMFA market exhibits a small but moderately time-varying response to changes in the opportunity cost; the calculation here suggests the mean elasticity of non-participants fluctuated between (absolute value) .07 and .22 between 1962 and 2005.

An important benefit of my approach lies in the ability to perform a very precise analysis of the connection between household behavior and aggregate money demand. I find that the elasticity of aggregated MA varies dramatically over the period, rising from a low of .15 in the early 1960s to a peak of .62 in 1981, before settling back to .22 by 2005. This behavior is induced by vast fluctuations of opportunity costs over the period. The aggregate elasticity is decomposed as the sum of contributions from the intensive margin of participants and non-participants, and an extensive margin of participation. While the intensive margin effects vary moderately over the sample period, and while the contribution of the extensive margin is never greater than 50% of aggregate elasticity, I find that most of the variability in aggregate MA elasticity stems from large fluctuations of the extensive margin contribution. This contribution varies from almost zero to .29 over the period.

The contribution of the extensive margin can be deconstructed further within the context of the theory. The intuition for its behavior may be gleaned as follows. Under a fixed costs regime, optimal behavior defines a cutoff of households current financial resources as a criterion for participation in NMFA markets. This threshold varies with idiosyncratic characteristics of households, but also with extant opportunity cost. When the opportunity cost is low, the threshold is relatively high; when the opportunity cost rises, the threshold falls. The principal determinant of the contribution of the extensive margin to aggregate MA elasticity is the location of the threshold with respect to the distribution of current resources in the population. Loosely speaking, if the threshold is in the heart of that distribution, then an increase
of the interest rate will induce a significant fraction of erstwhile non-participants to participate. Under the estimated parameterization of the model, the long upward spiral of the opportunity cost from the 1960s to 1981 pushed this threshold from irrelevance to a position making participation an important consideration for more households.

The rest of the paper is structured as follows. In the next section, I offer a sketch of the micro and macro evidence related to the goals of the model. In the third section, I present my model. The fourth section discusses some intuition for behavior in the model, and derives the decomposition of aggregate MA that I will use. The fifth section discusses the use of data and the estimation strategy. The results are reported in the sixth section, and the last section concludes.

2 Evidence

This section presents evidence related to the phenomena central to this paper. Although I will defer discussion of the details of the dataset until later, I will offer a brief introduction.

I utilize cross-sectional data on households and their behavior from the 1962 Survey of Financial Characteristics of Consumers (SFCC) and the 1983 and 1989 Survey of Consumer Finances (SCF). Time-series of economic aggregates are taken from the Federal Reserve Bank of Saint Louis’s FRED database. With respect to this data, I define “monetary assets” (MA) as the sum of balances of assets that are useful in making transactions, or are convertible after very little pecuniary or effort cost to such assets. In the cross-sectional data, the MA balances of a household is the sum of balances in checking accounts, savings accounts, and checkable money market accounts, plus an imputed value for holdings of currency.\footnote{Data on currency balances is not available, so an imputation procedure is devised that matches the ratio of currency to other MA constituents in the aggregate data to the analogous ratio of aggregated microdata. The details of this procedure will be discussed later.} I compare this microdata construct to the monetary aggregate M2M, that is, M2 minus small time-deposits; M2M consists mainly of currency, checking accounts, savings accounts, money market deposit accounts, and retail money market funds. The M2M own-rate is taken to be the applicable nominal rate of return on households’ MA holdings. “Non-monetary financial assets” (NMFA) is constituted of assets that are somewhat less liquid than MA, but that can be converted to MA at some cost. For households in the cross-sectional data, I take NMFA to be the sum of balances in non-checkable money-market mutual funds, certificates of deposit, other mutual funds, stocks, and
Table 1: Rates of participation in various asset classes in three years. Here, \( MA \) is the sum of all measured monetary assets; the notable exclusion in this measure is currency.

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>1962</th>
<th>1983</th>
<th>1989</th>
</tr>
</thead>
<tbody>
<tr>
<td>Checking Account</td>
<td>.594</td>
<td>.785</td>
<td>.812</td>
</tr>
<tr>
<td>Savings Account</td>
<td>.586</td>
<td>.617</td>
<td>.431</td>
</tr>
<tr>
<td>Checkable MM Accounts</td>
<td>NA</td>
<td>.104</td>
<td>.182</td>
</tr>
<tr>
<td>any MA</td>
<td>.771</td>
<td>.874</td>
<td>.853</td>
</tr>
<tr>
<td>CD</td>
<td>NA</td>
<td>.201</td>
<td>.197</td>
</tr>
<tr>
<td>Non-checkable MM Accts</td>
<td>NA</td>
<td>.053</td>
<td>.058</td>
</tr>
<tr>
<td>Stocks</td>
<td>.172</td>
<td>.191</td>
<td>.167</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>.027</td>
<td>.042</td>
<td>.056</td>
</tr>
<tr>
<td>Stock/Bond Mutual Funds</td>
<td>.049</td>
<td>.045</td>
<td>.072</td>
</tr>
<tr>
<td>any NMFA</td>
<td>.200</td>
<td>.359</td>
<td>.357</td>
</tr>
</tbody>
</table>

Table 1: Rates of participation in various asset classes in three years. Here, \( MA \) is the sum of all measured monetary assets; the notable exclusion in this measure is currency.

corporate bonds. I adopt the 5-year Treasury rate as the benchmark return for assets in this category. All of the data is deflated by the Consumer Price Inflation index normalized to the price level of 1982-84.

Table 1 shows the rates of participation of households in various financial instruments markets for 1962, 1983, and 1989. It can be seen that most households hold some form of MA besides currency, and it is assumed that all households utilize MA. In contrast, only 20% of households held any alternative financial instruments in 1962; this figure rises to 36% in 1983 and 1989.

Figure 1 shows the rate of participation of households in 20 quantiles of the distribution of real financial wealth plotted against the mean of log-wealth in each quantile; here, “financial wealth” refers to the sum of MA and NMFA balances of the household. Several features are noteworthy. First, it is clear that households with greater amounts of wealth are more likely to be participants. On the other hand, some households store substantial amounts of wealth in the form of MA; this is especially apparent for the earliest cross-section. Second, a shift is evident in behavior from 1962 to 1983; but the plots for 1983 and 1989 show little apparent difference. This is consistent with the evidence in Table 1.

Mulligan and Sala-i-Martin (2000) argue that the curves in Figure 1 suggest the

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\(^2\) All of the observations based on the SFCC and SCF data utilize the suggested frequency weights.
Figure 1: Fraction Adopting in Quantiles of the Log-Wealth Distribution. This figure shows the fraction of households holding positive amounts of NMFA (as defined in the text) in each 5% quantile of the distribution of financial wealth plotted against the log of the mean wealth in each quantile for each year.

In their model, households with higher levels of wealth, and thus greater potential benefit from a higher interest rate, are more likely to face participation costs that are lower than the benefit of participating.\footnote{A concise description of a version of Mulligan and Sala-i-Martin’s (2000) model is as follows. Assume that the fixed cost for household $i$ at date $t$ is $\psi_{it}$, a draw from some cumulative distribution function i.i.d. across households (but perhaps not across time). Furthermore, assume that the benefit of participating in NMFA markets for one period is proportional to the quantity of real wealth to be held as NMFA times the opportunity cost, say $\bar{r}_t$. Finally, assume that the fraction of wealth held as NMFA by participants is a constant, equal to $\alpha \in (0, 1)$, for all households. Under these assumptions, a household will choose to participate whenever $\psi_{it} \leq \bar{r}_t \alpha W_i$, where $W_i$ is household $i$’s real financial wealth, and each curve in Figure 1 can be interpreted as an empirical representation of the distribution of the statistic $\psi_{it} / (\bar{r}_t \alpha)$ for the given date.}
Figure 2: M2M-GDP Ratio and M2M Opportunity Cost. This figure plots the ratio of aggregate M2M to GDP, observed annually from 1959-2005, against the opportunity cost of M2M defined as the difference between the five-year Treasury bond rate and the M2M Own rate. Also shown are the best fit log-log (constant elasticity) curves for 1959-69 and 1977-99.
Figure 2 shows the relationship between the behavior of the aggregate M2M and the “M2M-opportunity cost”, essentially the spread between the 5-year Treasury rate and the M2M-own rate; the data is yearly from 1959-1999. Also shown is the best-fit log-linear curve that plays a prominent role in the empirical literature on aggregate money demand. The empirical phenomenon known as the “missing money” can be easily understood from this graph, as one imagines trying to predict the M2M-GDP ratio from pre-1969 data in the mid 1970s. The shift of the best-fit line also induces an increase of the implied sensitivity of money to changes in the opportunity cost after about the same transition in time; the elasticity inferred from the 1959-1969 data is −.17, while the 1977-1999 data suggest an elasticity of −.22.

Figure 2 depicts this evidence as a “velocity shift” (Reynard (2004)) and suggests a relationship to the opportunity cost, also shown.

3 Households’ Portfolio Choice Behavior

3.1 The Model

I consider the consumption and portfolio choices of a succession of cohorts of households each of whom lives for $N < \infty$ periods starting in the year of its “birth” (which is taken to correspond to “age” 26). In the presentation of the model, I adopt the point of view of a household $i$ who is born (i.e., aged $a = 26$) at date $\tau$ and dies after date $\tau + N - 1$. The household receives real income $Y^{i}_t$ at the beginning of each period while it is alive, where “real” quantities are denominated in units of contemporaneous consumption goods. The household allocates this income and real financial wealth held the beginning of the period to consumption of goods and accumulation of two types of financial assets. Consumption at $t$ is denoted $C^{i}_t$. The financial assets accumulated at $t$ are (real) monetary assets (MA) $M^{i}_t$ and (real) non-monetary financial assets (NMFA) $B^{i}_t$, and I take the convention that $M^{i}_t$ and $B^{i}_t$ are denominated in units of date $t + 1$ goods.

Holding MA between period $t$ and period $t + 1$ has a real cost of $\pi_t$ per unit; this cost may be attributed to inflation, but is offset by any interest earned by such assets. For example, if the rate of increase of the (nominal money) price of consumption is $\tilde{\pi}_t$, and the nominal rate of interest earned by MA is $i_t$, then $\pi_t$ is defined by the identity

$$1 + \pi_t \equiv \frac{1 + \tilde{\pi}_t}{1 + i_t}. \quad (1)$$

Similarly, NMFA held at the end of $t$ are assumed to earn real interest $r$ per unit at the beginning of date $t + 1$; so that, if $\tilde{\rho}_t$ is the nominal rate of return on NMFA, $r$
satisfies
\[
\frac{1}{1+r} = \frac{1 + \tilde{\pi}_t}{1 + \rho_t}.
\]  
(2)

Expressed in real terms, the budget constraint faced by the household is
\[
C^i_t + (1 + \pi_t) M^i_t + \frac{B^i_t}{1 + r} = Y^i_t + M^i_{t-1} + B^i_{t-1}.
\]  
(3)

It is assumed that \(\pi_t\) is a Markovian random variable realized at the beginning of period \(t\).\(^4\) The real interest rate earned by NMFA is assumed to be constant for simplicity. For convenience, I write
\[
H^i_t := Y^i_t + M^i_{t-1} + B^i_{t-1}
\]  
(4)

for the household’s current resources (analogous to what Deaton (1991) calls “cash in hand”). It is assumed that households are not allowed to issue MA or NMFA, so that households face liquidity constraints
\[
M^i_t, B^i_t \geq 0
\]  
(5)

while acting in the markets for each financial asset, as well.

For \(t < \tau + N\), household \(i\)’s date \(t\) (age \(26 + t - \tau\)) income \(Y^i_t\) satisfies
\[
\ln Y^i_t = \ln P^i_t + \ln u^i_t,
\]  
(6)

where \(\ln P^i_t\) and \(\ln u^i_t\) are stochastic components with the following properties. The permanent component of the household’s log-income is \(\ln P^i_t\), assumed to evolve for \(t > \tau\) as
\[
\ln P^i_t = \ln G(Z^i_t) + \ln P^i_{t-1} + \ln \nu^i_t.
\]  
(7)

where \(P^i_\tau\) is given at the time of the household’s birth; \(G(Z^i_t)\), the mean rate of growth of the household’s income, is function of a vector of idiosyncratic characterizstics \(Z^i_t\) that is assumed to be deterministic (as of the household’s birthdate); and \(\ln \nu^i_t\) is distributed normally with mean zero and variance \(\sigma^2_{\nu}(Z^i_t)\). The transitory component is \(\ln u^i_t\), assumed to be normal with mean zero and variance \(\sigma^2_u(Z^i_t)\). It is assumed that the permanent and transitory innovations \(\ln \nu^i_t\) and \(\ln u^i_t\) are jointly and serially independent over time and across households.

The function \(G\) and the variances of the innovations will be specified to capture

\(^{4}\)This assumption is consistent with a setting in which the price level at \(t + 1\) is observed at the beginning of period \(t\).
characteristics of log-income over the life-cycle as a function of the household’s age and other characteristics included in $Z_i^t$, as well as a level of technological progress that is taken to depend on the household’s birth cohort and education group. These details of the specification will be discussed in the Section 5.

Households have one unit of non-labor time that they allocate among leisure and activities related to making transactions and personal finance. First, as in McCallum and Goodfriend (1988), the household must spend time shopping for the goods it consumes. I assume that the shopping technology implies that the amount of time spent at $t$ depends on the ratio of consumption goods purchased to the amount of MA held; more precisely, the time spent shopping is equal to

$$\frac{\mu_1}{\mu_2} \left( \frac{C_i^t}{M_i^t} \right)^{\mu_2}.$$  

Second, accessing the bond market (i.e., setting $B_i^t > 0$) requires a fixed expenditure of time $X_i^t$ that depends on the household’s previous experience. To be more specific, I let $\xi_i^t \in \{0, 1\}$ denote an indicator of the previous experience of household $i$, defined recursively by $\xi_i^t = 0$ if $B_i^t = 0$ and

$$\xi_i^{t+1} = \begin{cases} 1 & \text{if } B_i^t > 0 \\ \xi_i^t & \text{if } B_i^t = 0 \end{cases}$$

for $t \geq \tau$. The fixed cost of participation is assumed to be equal to

$$X_i^t \equiv \mathbb{I} (\xi_i^t = 0) \ x_0 (Z_i^t) + x_1 (Z_i^t),$$

where $x_0, x_1 \geq 0$, and $\mathbb{I} (\cdot)$ is the indicator function. The function $x_1$ defines the amount of time required to use the NMFA market in each period after one has attained some previous experience, and $x_0$ is the incremental time cost incurred by inexperienced (i.e., first time) users. In sum, the leisure time of the household at $t$ is equal to $1 - Q_i^t$, where

$$Q_i^t \equiv \frac{\mu_1}{\mu_2} \left( \frac{C_i^t}{M_i^t} \right)^{\mu_2} + \mathbb{I} (B_i^t > 0) \ X_i^t.$$ 

The preferences of a household born (i.e., aged 26) at date $t$ are governed by a
value function of the form\footnote{Here and in what follows, the age subscripts attached to value functions of $Z^i_t$ are redundant under my assumption that $Z^i_t$ includes the age of the household. Such subscripts are attached for explicitness, and will be assumed to be consistent with the age implied by $Z^i_t$ when they are used.}

$$V_{26}(H^i_\tau, P^i_\tau, \xi^i_\tau, \pi_\tau, Z^i_\tau) = E_\tau \left\{ \sum_{s=\tau}^{\tau+N-1} \beta^{s-\tau} \left[ \ln C^i_s + \ln (1 - Q^i_s) \right] \right\}, \quad (12)$$

where $E_\tau (\cdot)$ is the mathematical expectation operator over random variables

$$\{\pi_{\tau+s}, P^i_{\tau+s}, u^i_{\tau+s} \}_{s=1}^{N-1} \quad (13)$$

conditional on the value $\pi_\tau$. The problem of the household is then to choose stochastic processes for consumption and a portfolio allocation to maximize (12) subject to (3)-(11) taking $(H^i_\tau, P^i_\tau, \xi^i_\tau, \pi_\tau), \{Z^i_s\}_{s=\tau}^{\tau+N-1}$, and the processes generating the random variables (13) as given.

### 3.2 Characterizing Optimal Decision Rules

The assumptions on preferences and incomes imply that the household’s objective function and the budget constraints are each homogeneous in the permanent component of income. Since the stochastic elements of the environment are Markovian, optimal behavior may conveniently be characterized with reference to a recursive characterization of normalized consumption and portfolio choice. I use lower-case letters to denote model quantities normalized by the permanent component of income, i.e.,

$$h^i_t \equiv \frac{H^i_t}{P^i_t}, \quad (14)$$

e tc. Now the “transformed problem” of a household of age $a$ at $t$ is defined to be

$$v_a(h^i_t, \xi^i_t, \pi_t, Z^i_t) = \max_{c, m, w \geq 0} \left[ \ln c + \ln (1 - q) + \beta E_t v_{a+1}(h', \xi', \pi', Z^i_t) \right] \quad (15)$$

subject to

$$c + (1 + \pi_t) m + \frac{w - m}{1 + r} = h^i_t \quad (16)$$

$$h' = u' + \frac{w}{G(Z_{t+1})} \nu' \quad (17)$$
and

\[ w \geq m, \]  

where

\[ \xi' = \begin{cases} 
1 & \text{if } w > m \\
\xi_i & \text{if } w = m 
\end{cases} \]  

and

\[ q = \frac{\mu_1}{\mu_2} \left( \frac{c}{m} \right)^{\mu_2} + \mathbb{I}(w > m) X_i^j. \]  

The Bellman equation is valid for \( a \in \{26, \ldots, 26 + N - 1\} \), and the value function after the last period is

\[ v_{26+N}(h_i^t, \xi^i_t, \pi_t, Z_i^t) \equiv 0. \]  

Note that I have replaced the choice of next period’s MA and NMFA by the equivalent choice of (normalized) MA and total financial assets \( w_i^t \equiv m_i^t + b_i^t \), so that the condition \( b_i^t \geq 0 \) is equivalent to \( w_i^t \geq m_i^t \), and participation in the bond market is indicated by \( w_i^t > m_i^t \). For ease of notation, I have also replaced the symbols \( \pi_{t+1}, \nu_{t+1}^i \) and \( u_{t+1}^i \) by \( \pi', \nu', \) and \( u' \), respectively; I will follow this convention below, as well, where there can be no confusion.

The programs specified by the definitions of the functions \( v_a(h, \xi, \pi, Z) \) are solved by functions \( m_a(h, \xi, \pi, Z), w_a(h, \xi, \pi, Z), \) with the functions \( c_a(h, \xi, \pi, Z) \) and \( b_a(h, \xi, \pi, Z) \) defined in the obvious way, for each \( a \). I also define \( \chi_a(h, \xi, \pi, Z) \) to be an indicator for the decision to participate in the NMFA market; that is, \( \chi_a(h, \xi, \pi, Z) = 1 \) if \( m_a(h, \xi, \pi, Z) > w_a(h, \xi, \pi, Z) \) and \( \chi_a(h, \xi, \pi, Z) = 0 \) otherwise.

As no closed form solution exists for these functions, they must be approximated numerically. The numerical implementation is discussed in more detail in the Appendix.

4 Preliminary Analysis

4.1 Understanding Household Portfolio Choice

Let us begin by analyzing the behavior of a household that optimally chooses to participate in the NMFA mkt. This point of view presumes that the liquidity constraint is non-binding; and that either there are no applicable fixed costs, or the household (optimally) absorbs the fixed costs in order to participate. The first-order
conditions for consumption and MA balances for this household are (respectively)

\[ \frac{1}{c} \left[ 1 - \left( \frac{\mu_1}{1 - q} \right) \left( \frac{c}{m} \right)^{\mu_2} \right] - \lambda = 0 \]  

(22)

and

\[ \frac{\mu_1}{c (1 - q)} \left( \frac{c}{m} \right)^{1+\mu_2} - \lambda \tilde{r}_t = 0, \]  

(23)

where \( \lambda \) is a non-negative Lagrange multiplier on the budget constraint, and

\[ \tilde{r}_t \equiv 1 + \pi_t - \frac{1}{1 + r}. \]  

(24)

(The constraint that MA balances must be non-negative may be ignored under the assumption that \( \mu_1 \) and \( \mu_2 \) are strictly positive, as I will maintain throughout this analysis.) For an NMFA market participant, \( \tilde{r}_t \) is the marginal rate of transformation between MA and consumption holding total wealth \( w \) constant; henceforth, I will refer to \( \tilde{r}_t \) as the “opportunity cost” of holding MA balances, despite the fact that this label is apt only for participants.

Substituting from (22), (23) can be written as

\[ \frac{\mu_1}{(1 - q)} \left( \frac{c}{m} \right)^{1+\mu_2} \left[ 1 - \left( \frac{\mu_1}{1 - q} \right) \left( \frac{c}{m} \right)^{\mu_2} \right]^{-1} = \tilde{r}_t. \]  

(25)

Thus the marginal rate of substitution of MA balances for consumption, the left-hand side of (25), is equal to the opportunity cost of MA balances \( \tilde{r}_t \). The implications of this static trade-off may be interpreted by comparison to the conventional theory of the demand for money following Baumol (1952) and Tobin (1956). In their analyses, holding higher money balances reduces the number of costly withdrawals that must be made to effect a given volume of transactions; and the “opportunity cost” arises from the foregone interest earnings of higher yielding alternatives to money. In addition to the presence of liquidity constraints and participation costs, different implications of the present model stem from the fact that both the volume of the transactions and the cost of the time spent in making them are elastic to the quantity of MA balances.

On the other hand, for NMFA market participants, at least, this difference may be small. To see this, use (20) to eliminate \( q \) and derive

\[ \left( \frac{c}{m} \right)^{1+\mu_2} + \tilde{r}_t \left( 1 + \frac{1}{\mu_2} \right) \left( \frac{c}{m} \right)^{\mu_2} - \frac{\tilde{r}_t (1 - X_i)}{\mu_1} = 0, \]  

(26)
where one should recall that $X^i_t$ is the fixed cost incurred by the household by participating. Now it can be seen that, if $\tilde{r}_t (1 + 1/\mu_2)$ and $X^i_t$ are small (relative to 1), we have approximately

$$m \approx c \left( \frac{\mu_1}{\tilde{r}_t} \right)^{1+\mu_2}.
$$

(27)

Now to the extent that consumption is invariant to changes in the opportunity cost, the opportunity cost elasticity of MA balances is approximately equal to $-(1 + \mu_2)^{-1}$. Consistent with conventional theories of consumption and saving, we will find that consumption is relatively inelastic whenever MA balances constitute only a small component of lifetime resources. The last property will be seen to hold whenever the household participates in the NMFA market, since wealth accumulation is mostly divorced from maintenance of transactions balances. These findings suggest that an inventory theoretic approach balancing the marginal benefit in shopping against the opportunity cost of foregone interest may explain this aspect of the behavior of NMFA market participants well. In particular, (27) is exactly the formula derived by Baumol (1956) when $\mu_2$ is equal to 1.

The static trade-off interpretation and the comparison to the Baumol-Tobin model dissolve when the liquidity constraint is binding, or when the household is induced not to participate because of fixed costs. This is because the choice of MA balances is inseparable from the accumulation of wealth across time when the household chooses not to participate in the NMFA market. Taking first the case of a binding liquidity constraint, (23) becomes

$$\frac{\mu_1}{c (1-q)} \left( \frac{c}{m} \right)^{1+\mu_2} - \lambda \tilde{r}_t - \phi = 0,
$$

(28)

where $\phi > 0$ is a Lagrange multiplier on the liquidity constraint. Now from the result analogous to (25), it follows that the marginal rate of substitution between MA balances and consumption is greater than the opportunity cost. In this case, households would like to obtain a larger volume of MA balances by borrowing in the NMFA market, even at an interest premium of $\tilde{r}_t$ over the rate of return on MA balances, but they are prevented from doing so.

Figure 3 illustrates the properties of the optimal policy functions for (normalized) consumption, MA balances, and NMFA balances for a household that faces no fixed costs of participation at a fixed (moderate) value of the opportunity cost $\tilde{r}_t$. More precisely, the policy functions shown are those of an “experienced” household in a parameterization of the model in which the per-period component of the fixed cost
$x_1$ is equal to zero. For low levels of current resources, this household is liquidity constrained, and chooses not to participate in the NMFA market. Note that a positive quantity of MA balances is required at all levels of current resources to effect consumption transactions. As current resources rise, so do MA balances. At some threshold level of current resources, the liquidity constraint ceases to bind and the household begins to participate in the NMFA market. Intuitively, this threshold defines the point at which the motivation to accumulate wealth \textit{per se} overtakes the need to maintain transactions balances. In what follows, I will refer to this point as the “transactions-saving threshold”.

The locus of the consumption policy function bears resemblance to that in models

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6The policy functions shown in Figures 3 and 4 are those of a 45-year-old high-school-educated household under the parameterization estimated for the start–up costs model (SC) specified later in the paper. The opportunity cost is set at $\tilde{r}_t = 3.83\%$ in each case.
of buffer-stock or precautionary saving.\textsuperscript{7} In particular, infinite marginal utility at zero consumption in combination with liquidity constraints induce the very high marginal propensity to consume out of current resources at low levels of the latter; and the MPC moderates at levels of current resources such that the liquidity constraint is no longer binding. In these models, households maintain a positive “buffer-stock” of financial wealth even when their current resources are low to insure themselves against the possibility of a very low income realization in the subsequent period. Here, that motivation is augmented by the need to maintain transactions balances. As a result, the buffer-stock balance is somewhat higher, and is constituted entirely of MA at low levels of current resources. It should be noted that, absent the value of money in making transactions, this household would hold no MA balances at any level of current resources.

Conceptually, it is clear how the model endogenously generates non-participants even abstracting from fixed costs of participation; since households cannot perfectly insure themselves against income shocks, there will always be households whose current resources fall below the transactions-saving threshold. Moreover, even households that participate in the NMFA market may hold substantial MA balances for use in making transactions. The last property, observed also in the data, would not be exhibited in a model of portfolio choice without the transactions motive when the monetary asset is dominated in rate of return. On the other hand, looking closely at Figure 3 may lead one to doubt that this version of the model will suffice to explain high ratios of MA balances to income frequently observed in the data.

Let us now turn to the special issues faced by a household facing positive fixed costs of participation. This household may be viewed as solving two problems, defined by the choice to participate or not to participate, and then comparing the utility values implied by contingent optimal behaviors. Conditional on the choice not to participate, we may interpret the first-order conditions for consumption and MA balances to be (22) and

\[
\frac{\mu_1}{c(1-q)} \left( \frac{c}{m} \right)^{1+\mu_2} - \lambda \tilde{r}_t - \phi + \psi = 0 \tag{29}
\]

where the element distinguishing (29) from (28) is a non-negative Lagrange multiplier on the constraint \( w \leq m \) that applies when the household chooses not to participate. Of course, the liquidity constraint cannot bind simultaneously with this new constraint. Apparent from (22) and (29) is the fact that the marginal rate of

\textsuperscript{7}Important papers in this literature include Carroll and Samwick (1997) and Gourinchas and Parker (2002).
substitution of MA balances for consumption is lower than \( \bar{r}_t \) when the household is induced not to participate by positive fixed costs (that is, when \( \phi = 0 \) and \( \psi > 0 \)).

Figure 4 shows the optimal policy functions for a household facing positive fixed costs of participation in the NMFA market. Satisfying intuition, the value of participation rises with the level of current resources, so that the choice to participate again follows a threshold rule. The decomposition of the problem, however, suggests a conceptual distinction between the transactions-saving threshold analyzed previously, and a threshold for participation in the present circumstance. The new threshold, which I will refer to as the “participation threshold”, is defined by the infra-marginal decision to participate, which requires that the value of participation exceeds the disutility associated with the fixed cost. When the household faces no fixed costs, as in Figure 3, the two thresholds coincide. With positive fixed costs, however, crossing the lower threshold is no longer sufficient to induce participation.

With the introduction of the second threshold, the set of values of current re-
sources over which the household will refrain from using the bond market is extended. Thus, in the sense of fitting the data, the fixed cost is a tool for shaping the behavior of households under the model. Of course, the distribution of households’ current resources is endogenous and depends upon these policy functions, so the exercise cannot be conducted analytically.

4.2 The Response of the Participation Threshold to a Change of the Opportunity Cost

In this subsection, I will examine more closely the effect of a change of the opportunity cost on the participation threshold. As defined in (24), the opportunity cost moves one-for-one with the cost of holding MA balances \( \pi_t \); thus, I will speak in the rest of the paper of a response to changes of the opportunity cost when analyzing the effect of a change of \( \pi_t \) in the model. Moreover, these changes in the cost of holding MA balances may be interpreted to result from a change in the rate of inflation.

Figures 5 and 6 show the policy functions for the same households considered in Figures 3 and 4, except that two different values of the opportunity cost are considered. For the case of the experienced household in Figure 5, note first that the consumption policy locus changes very little in response to the change of the opportunity cost. For households with current resources below the transactions-saving threshold, i.e., households that are liquidity constrained, there is little adjustment of MA balances, either. This is consistent with the small role of MA in saving in comparison with its importance in effecting transactions in this region. We may infer that the marginal rate of substitution of MA balances for consumption is little affected, and this suggests that the cost of the shock in terms of welfare may be small for households below the transactions-saving threshold.

Above the transactions-saving threshold, experienced households appear to make significant adjustments between MA and NMFA in response to the change of the opportunity cost. When the opportunity cost rises, MA balances fall and NMFA holdings rise. Considered in conjunction with the lack of adjustment of consumption, this verifies that these households adjust in a manner that is well-described by the static trade-off analysis of the previous subsection.

The case of inexperienced households, shown in Figure 6, is distinguished in several important ways. First, between the transactions-saving threshold and the participation threshold, we see a more significant response of consumption at the margin. This fact stems from the fact that MA balances constitute a non-negligible component of lifetime resources in this region. Second, a rise of the opportunity cost reduces the participation threshold, since the value of participation is increased at
Figure 5: Policy functions of an experienced households for different values of opportunity cost, 3\% and 3.6\% (a 20\% increase). The lower MA locus, and the higher NMFA locus correspond to the higher value of the opportunity cost.
Figure 6: Policy functions of an inexperienced household for two values of opportunity cost, 3% and 3.6% (a 20% increase). The lower MA locus, and the higher NMFA locus correspond to the higher value of the opportunity cost.
each level of current resources. This effect is especially pronounced for the case of start-up costs, since inflation (and the opportunity cost) is highly persistent, and the value of future participation weighs importantly in the calculation. As will be seen, this variation of the participation threshold induces an important component in the response of aggregate MA balances to these changes.

4.3 Implications for the Elasticity of the Monetary Aggregate

Suppressing the role of indiosyncratic characteristics, let us write \( \omega_{a,t}(H, P, \xi) \) for the distribution of households’ distinguishing characteristics (ages \( a \), current resources \( H \), market experience \( \xi \), and permanent components of income \( P \)) conditional upon the realized sequence of the cost of holding MA balances \( \{\pi_s\}_{s=-\infty}^t \). Then aggregate MA balances can be represented as

\[
M_t = \sum_{a=1}^N \sum_{\xi \in \{0,1\}} \int_P \int_H M_a(H, P, \xi, \pi_t) \chi_a \left( \frac{H}{P}, \xi, \pi_t \right) \omega_{a,t}(H, P, \xi) \, dHdP
\]

\[
+ \sum_{a=1}^N \sum_{\xi \in \{0,1\}} \int_P \int_H M_a(H, P, \xi, \pi_t) \left[ 1 - \chi_a \left( \frac{H}{P}, \xi, \pi_t \right) \right] \omega_{a,t}(H, P, \xi) \, dHdP,
\]

where \( M_a(H, P, \xi, \pi_t) \equiv m_a \left( \frac{H}{P}, \xi, \pi_t \right) P \) and the presence of the indicator \( \chi_a(h, \xi, \pi_t) \) makes explicit the sum of the components of participants and non–participants. Working under the assumption that each of these functions is continuously differentiable except (possibly) at the participation threshold, we have

\[
\frac{\partial M_t}{\partial \pi} = \sum_{a=1}^N \sum_{\xi \in \{0,1\}} \int_P \int_H \frac{\partial M_a(H, P, \xi, \pi_t)}{\partial \pi} \chi_a \left( \frac{H}{P}, \xi, \pi_t \right) \omega_{a,t}(H, P, \xi) \, dHdP
\]

\[
+ \sum_{a=1}^N \sum_{\xi \in \{0,1\}} \int_P \int_H \frac{\partial M_a(H, P, \xi, \pi_t)}{\partial \pi} \left[ 1 - \chi_a \left( \frac{H}{P}, \xi, \pi_t \right) \right] \omega_{a,t}(H, P, \xi) \, dHdP
\]

\[
- \sum_{a=1}^N \sum_{\xi \in \{0,1\}} \int_P \Delta_a \left( \hat{H}_a(P, \xi, \pi_t), \xi, \pi_t \right) \frac{\partial H_a(P, \xi, \pi_t)}{\partial \pi} \omega_{a,t}(\hat{H}_a(P, \xi, \pi_t), P, \xi) \, dP,
\]

where

\[
\Delta_a \left( \hat{H}, \xi, \pi_t \right) \equiv \lim_{H \uparrow \hat{H}} M_a(H, P, \xi, \pi_t) - \lim_{H \downarrow \hat{H}} M_a(H, P, \xi, \pi_t)
\]
is the change of MA balances across the value $H$, and $H_a(P, \xi, \pi_t)$ is the value of current resources at the participation threshold. In (30), the first two components represent the contribution to the derivative of the behavior of participants and non-participants, respectively; and the third component represents the contribution of households that would be induced to change their participation status if the cost of holding money were to change. Now it can be seen that the elasticity of this aggregate is

$$E_t = \sum_{a=1}^{N} \sum_{\xi \in \{0,1\}} \int P H \left( e_a \left( \frac{H}{P}, \xi, \pi_t \right) \left( \frac{M_a(H, P, \xi, \pi_t)}{\mathcal{M}_t} \right) \chi_a \left( \frac{H}{P}, \xi, \pi_t \right) \omega_{a,t} (H, P, \xi) dH dP \right)$$

$$+ \sum_{a=1}^{N} \sum_{\xi \in \{0,1\}} \int P H \left( e_a \left( \frac{H}{P}, \xi, \pi_t \right) \left( \frac{M_a(H, P, \xi, \pi_t)}{\mathcal{M}_t} \right) \left[ 1 - \chi_a \left( \frac{H}{P}, \xi, \pi_t \right) \right] \omega_{a,t} (H, P, \xi) dH dP \right)$$

$$- \sum_{a=1}^{N} \sum_{\xi \in \{0,1\}} \int P H \left( \bar{e}_a (\xi, \pi_t) H_a (P, \xi, \pi_t) \left( \frac{\Delta_a (H_a(P, \xi, \pi_t), \xi, \pi_t)}{\mathcal{M}_t} \right) \omega_{a,t} (H_a(P, \xi, \pi_t), P, \xi) dP \right)$$

where $e_a(h, \xi, \pi_t)$ is a household’s opportunity cost-elasticity of MA balances, and $\bar{e}_a(\xi, \pi_t)$ is the elasticity of the participation threshold for a household with experience $\xi$ when the cost of holding MA is $\pi_t$. Note that the terms of the form $M_a(H, P, \xi, \pi_t) / \mathcal{M}_t$ in the integrands of the first two terms may be interpreted as the shares of individual households in total MA balances, so that these terms are means over households’ MA elasticities weighted by these shares.

Let us look more closely at the least familiar element of this decomposition, which captures the effect of the extensive margin. Defining

$$\bar{\omega}_t \equiv \sum_{a=1}^{N} \sum_{\xi \in \{0,1\}} \int_P \omega_{a,t} (H_a(P, \xi, \pi_t), P, \xi) dP$$

to be the mean density of the distribution of households at the participation thresh-
old, write
\[ \tilde{\omega}_{a,t}(P, \xi) = \frac{\omega_{a,t}(\tilde{H}_a(P, \xi, \pi_t), P, \xi)}{\tilde{\omega}_t}, \]
so that the term in question can be conveniently written as
\[
\tilde{\omega}_t \left\{ \tilde{E}_{a,t} \left[ \hat{e}_a(\xi, \pi_t) \tilde{H}_a(P, \xi, \pi_t) \right] \tilde{E}_{a,t} \left[ \frac{\Delta_a(\tilde{H}_a(P, \xi, \pi_t), \xi, \pi_t)}{\mathcal{M}_t} \right] \right. \\
+ \tilde{\text{Cov}}_{a,t} \left\{ \left[ \hat{e}_a(\xi, \pi_t) \tilde{H}_a(P, \xi, \pi_t) \right], \left[ \frac{\Delta_a(\tilde{H}_a(P, \xi, \pi_t), \xi, \pi_t)}{\mathcal{M}_t} \right] \right\}, \tag{31}
\]
where \( \tilde{E}_{a,t}(\cdot) \) and \( \tilde{\text{Cov}}_{a,t} \) denote the mean and covariance with respect to the distribution defined by \( \tilde{\omega}_{a,t}(P, \xi) \).

The objects in this decomposition of the extensive margin effect may be ascribed economic content. While \( \tilde{\omega}_t \) is on one level simply an integrating constant, it is simultaneously a measure of how relevant the decision to participate is within the population. \( \tilde{E}_{a,t} \left[ \hat{e}_a(\xi, \pi_t) \tilde{H}_a(P, \xi, \pi_t) \right] \) maybe interpreted as the mean of the change of the participation threshold induced by a 1% change of the opportunity cost across households. Similarly, \( \tilde{E}_{a,t} \left[ \Delta_a(\tilde{H}_a(P, \xi, \pi_t), \xi, \pi_t) / \mathcal{M}_t \right] \) is the mean of the drop of MA balances as a household’s level of current resources passes the threshold.

Looking at Figure 7 may assist with the intuition. There, the optimal MA balances policy loci for two households is shown superimposed upon the (endogenous) distribution of normalized current resources in the population.\(^{10}\) To fix ideas, these are the policies relevant to the household at the values of the opportunity cost extant in 1962 and 1983. Intuitively, the threshold is not relevant for a large fraction of households in 1962; but is is much more in the body of the distribution in 1983. the “relevance” is captured by the \( \tilde{\omega}_t \) terms, while the mean of the change of the threshold measures it’s sensitivity to small changes of the opportunity cost. On the other hand, crossing the threshold in 1962 implies a much greater drop of MA balances for the household than it would to cross the threshold in 1983; this effect is captured by the last object in the decomposition.

This decomposition of the aggregate elasticity informs a discussion of how the conventional analysis may fail. First, if there is significant heterogeneity in the response of individual households, then an analysis based on aggregates will inaccurately measure the sum of the first terms. More significant, perhaps, is the fact

\(^{10}\)For clarity in the figure, the distribution of current resources is represented as the lognormal approximation of the actual distribution.
Figure 7: MA balances policy functions at opportunity costs extant in 1962 (1.61%) and 1983 (5.4%). The distribution of normalized current wealth for 1962 has been superimposed.
that the third term is entirely ignored by the static-tradeoff approach. The ability to implement this accounting for the elasticity of aggregate MA elasticity is the major benefit of the approach taken here.

The detailed dissection of the effect of the extensive margin will facilitate a very precise accounting for the time series of aggregate MA balances in what follows.

5 Empirical Implementation

5.1 Parameters Calibrated to Micro-Data

Cross-sectional (household) data that will be used in calibration and estimation exercises come from the 1962 Survey of Financial Characteristics of Consumers (SFCC) and the 1983 and 1989 Surveys of Consumer Finances (SCF). The SCF are the institutional successors of the SFCC, and are thus similar and largely compatible. Hereafter, I will sometimes refer to the 1962 SFCC and the SCF surveys loosely as SCF surveys.\footnote{These years are chosen for the high degree of variability of interest rates over the period, and includes a peak of interest rates and a subsequent moderation. As I discuss below, these properties induce an observable hysteresis effect that facilitates identification of start-up costs.}

As noted above, “birth” in the model will be taken to correspond to the age 26 in the data, and I take $N = 90$ to be the last year of life. The terminal year has been chosen to be long enough to impart sufficient consideration of the future to households near the retirement age of 65 without introducing an ad hoc value of bequeathing wealth after death.

I use the consumer price index (CPI) for all urban consumers from the U.S. Bureau of Labor Statistics to deflate financial wealth and income in the data to comparable quantities. Real quantities in the paper are normalized to 1983 prices.

5.1.1 Households’ Portfolio Balances

Table 1 in Section 2 displays some statistics on households’ participation in markets for various asset classes from the SCF data. To compare the implications of the model to the data, MA is defined as the sum of currency holdings, checking accounts, savings accounts, money market deposit accounts, and checkable money market mutual fund accounts; and NMFA is taken to be the sum of balances in non-checkable money market mutual funds, certificates of deposit, other mutual funds, stocks, and corporate bonds. A household is defined to participate in the NMFA market if the
measure of NMFA defined above is observed as positive. A household’s total wealth is defined to be the sum of MA and NMFA balances.

Except for currency holdings, construction of these measures for individuals in the SCF data is straightforward. Data on households’ holdings of currency is not available in the data, however. As the model developed in the previous section makes predictions only for holdings of an aggregated measure of “money”, it is necessary to impute a value for households currency holdings to fill the gap. This is accomplished as follows.

I specify parametrically that

\[ \text{CURR}_t^i = a_t \log \left( 1 + \text{MA}_t^i \right) + b_t \log \left( 1 + \hat{Y}_t^i \right), \]

where \( \text{CURR}_t^i \) is a household’s currency balances; \( \text{MA}_t^i \) is total balances of other (i.e., observed) MA components; \( \hat{Y}_t^i \) is augmented income defined by

\[ \hat{Y}_t^i = \tilde{Y}_t^i + c_t \text{NMFA}_t^i; \]

\( \tilde{Y}_t^i \) is income (as above); \( \text{NMFA}_t^i \) is measured NMFA; and \( a_t, b_t, \) and \( c_t \) are constants across the population for each year. To provide some intuition for this choice, I start from the presumption that MA in some form is optimally retained in increasing quantities to accommodate increasing consumption. Since I don’t observe consumption, \( \hat{Y}_t^i \) provides a proxy that (like consumption) is higher when income and wealth are higher. When \( b_t > 0 \), currency holdings increase in this proxy for consumption. On the other hand, the need to hold currency is offset by holdings of other forms of MA; this is captured in the formulation above when \( a_t < 0 \). Note also that, under these parametric restrictions, currency holdings approach zero as the ratio \( \left( 1 + \text{MA}_t^i \right) / \left( 1 + \hat{Y}_t^i \right) \) goes to infinity; thus, if the household holds a lot of money in non-currency forms, the currency holdings imputed to the household will be small. On the other hand, currency holdings approach \( b_t \log \left( 1 + \hat{Y}_t^i \right) \) as \( \text{MA}_t^i \) goes to zero, so that this is exactly the quantity of currency imputed to households that hold no other forms of MA.

The parameter \( c_t \) is a proxy for the marginal propensity to consume out of NMFA; given the impact of the choice on the results, there should be little controversy in setting this value to 0.05. The parameters \( a_t \) and \( b_t \) are chosen to meet the following criteria at each \( t \). First, I match the ratio of the sum of the currency holdings of the households in the microdata to the sum of their holdings of other monetary assets to the analogous ratio in the data; the appropriate target statistics are tabulated in
<table>
<thead>
<tr>
<th>Year</th>
<th>Curr (1)</th>
<th>Check (2)</th>
<th>Svgs (3)</th>
<th>RMF^{12} (4)</th>
<th>(1)/[(2)+(3)+(4)]</th>
</tr>
</thead>
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<td>1962</td>
<td>29.82</td>
<td>116.22</td>
<td>67.57</td>
<td>0</td>
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</tr>
<tr>
<td>1983</td>
<td>140.09</td>
<td>236.99</td>
<td>342.49</td>
<td>143</td>
<td>.1937</td>
</tr>
<tr>
<td>1989</td>
<td>217.33</td>
<td>279.83</td>
<td>526.58</td>
<td>280.9</td>
<td>.1999</td>
</tr>
</tbody>
</table>

Table 2: Components of M2M (Billions of dollars).

<table>
<thead>
<tr>
<th>Year</th>
<th>$a_t$</th>
<th>$b_t$</th>
<th>$c_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1962</td>
<td>-.3113</td>
<td>.8816</td>
<td>.05</td>
</tr>
<tr>
<td>1983</td>
<td>-.2984</td>
<td>.9031</td>
<td>.05</td>
</tr>
<tr>
<td>1989</td>
<td>-.2856</td>
<td>.9316</td>
<td>.05</td>
</tr>
</tbody>
</table>

Table 3: Parameters of the currency imputation.

Figure 2. Second, I set the ratio of the sum of the currency holdings of households who hold no other MA to the sum of their $\bar{Y}_t$ equal to two times the same ratio for the households with positive measured MA; this captures the intuitively appealing idea that households that operate without other forms of money than currency must hold more currency than other households. The resulting parameters are shown in Figure 3.

Households’ MA balances are accordingly defined as the measured quantity plus these imputed currency balances. The imputations augment by 16%, 19%, and 20% the aggregate of households’ measured MA for 1962, 1983, and 1989 (respectively). The measure of MA for households is comparable to the measure of aggregate money stock called M2M, or M2 minus small time deposits. Reynard (2004) focusses his analysis on this measure. The measure differs from that used by Mulligan and Sala-i-Martin (2000) by the addition of money market deposit accounts and checkable money market mutual funds. The idea is that these assets, while they may earn some small interest, have negligible participation costs, and they are readily useful in transactions.

5.1.2 Households’ Labor Incomes

In the SCF data, households’ exogenous incomes are measured as the sum of the following components: wages; salaries; net incomes from sole proprietorships, professional partnerships, businesses, and farms; and other forms of non-financial income, including unemployment and worker’s compensation, child support, alimony, and payments from various forms of welfare programs. I include also pension and retirement income and social security income. Next, I obtain measures of after-
tax labor incomes by netting out estimates of the taxes paid by each household as constructed by NBER’s TAXSIM software. I drop households with zero reported income. Finally, in order to use the innovation variance estimates of Cocco, Gomes, and Maenhout (2002), as discussed below, I also drop households with female heads.

It is well-known in the literature that income age-profiles differ significantly by the household’s level of education.\footnote{See, e.g., Hubbard, Skinner, and Zeldes (1994) or Attanasio (1995).} I follow Cocco, Gomes, and Maenhout (2002) in identifying three levels of education: households with less than a complete high school education (“no degree”); those who have completed high school, but have not completed College (“high school graduates”); and those who have earned a college degree (“college graduates”).

To complete the specification of the previous section, I need to state the nature of the initial shocks $P_i$, where $\tau$ is the birth year of the household, the deterministic processes $G(Z_i)$, and the variances $\sigma^2_v(Z_i)$ and $\sigma^2_u(Z_i)$.

Specification of the labor income process is meant to control for the predictable effects of households’ idiosyncratic characteristics, while leaving a stochastic component close to that actually experienced by the households. To accomplish this, I begin by assuming that the mean of the stochastic log-income of household $i$ at $t$ is given by

$$\ln F_i = f(AGE_i, BYR_i, EDUC_i).$$

Here, $AGE_i$ is the age of household $i$, $BYR_i$ is the birth year of the household, and $EDUC_i$ is the education group of the household. It is further assumed that $EDUC_i$ is fixed for $i$ by the time the household achieves age 26.

Next, specify that, for household $i$ born in year $\tau$,

$$\ln P^i_\tau = \ln F^i_\tau + \ln \nu^i_\tau$$

and for $t > \tau$,

$$\ln G^i_t = \ln F^i_t - \ln F^i_{t-1}.$$

Within each education group, I set the variance of the initial shock to the permanent component $(\sigma^2_{\nu, \tau})^2$ equal to the variance of log-income of 24-28 year-olds minus the variance of the transitory component, so that the dispersion matches that in the data. For subsequent years up to retirement, I use the estimates of Cocco, Gomes, and Maenhout (2005), based on PSID data, to calibrate the variances of the income process innovations within each education group.

I assume that households beyond the age of 65 face a transitory shock to net income related to uncertainty in healthcare and other expenditures. Formally, I set
\[ \sigma_0^2 (Z_{it}) \quad \sigma_\nu^2 (Z_{it}) \quad t > \tau \quad \sigma_\nu^2 (Z_{it}) \]

<table>
<thead>
<tr>
<th></th>
<th>( \sigma_0^2 (Z_{it}) )</th>
<th>( \sigma_\nu^2 (Z_{it}) ), ( t &gt; \tau )</th>
<th>( \sigma_\nu^2 (Z_{it}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No H.S. degree</td>
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<td>.0105</td>
<td>.2730</td>
</tr>
<tr>
<td>H.S. Grad</td>
<td>.0738</td>
<td>.0106</td>
<td>.2891</td>
</tr>
<tr>
<td>College Grad</td>
<td>.0584</td>
<td>.0169</td>
<td>.2273</td>
</tr>
</tbody>
</table>

Table 4: Growth rates and stochastic properties of households' income processes.

\( \sigma_\nu^2 (Z_{it}) = 0 \) and \( \sigma_\nu^2 (Z_{it}) = 0.10 \) for households older than 65.

The results are summarized in Table 8 and Figure 21 below. The table reports the dispersion characteristics, while the figure shows the mean of income for each age and education group within the years 1962 and 1989 implied by the estimated process; the profile for 1983 is excluded so that the curves may be more easily discerned. Details of the construction of these processes are relegated to an appendix.
5.1.3 Initial Wealth Distribution

Simulation of the model relies on specification of the ratio of wealth to the permanent component of income at the date of birth. This distribution is assumed to be log normal with parameters estimated based on the ratio of total financial wealth to income in the SCF data for households aged 24-28 (following Gourichas and Parker (2002)). I allow the distribution parameters to vary with the household’s education. The distribution in the data shows little tendency to evolve over time, and so it is viewed that a constant distribution is a reasonable calibration. The follow table summarizes the estimates from the data as used in the calibration.

5.2 Calibration of Aggregate Processes

Simulation of the model requires a specification of the process that determines the cost of holding MA $\pi_t$, as well as input of values for the (constant) real interest
rate $r$ and the realized time series $\{\pi_t\}$. To simulate data on 65 year olds in 1962 and 26 year olds in 1989, I require this data for the years 1923 to 1989.

I begin by constructing a measure of price inflation using the CPI series, which is available from 1921 from the U.S. Bureau of Labor Statistics.

I adopt the yield on the 5-year Treasury Notes as a proxy for the nominal rate of return on NMFA. While the three-month T-Bill rate is a common proxy for the opportunity cost of holding assets in M1 money stock in the empirical money demand literature, I view this as too limited to stand for the group of assets I have in mind. The 5-year rate may be thought to capture the return and risk properties of NMFA in the model.

While data on the 3-month bill is available from the Federal Reserve Board of Governors from 1921, the 5-year T-Note yield is available (from FRED) only from 1953. To impute value for earlier periods, I assume that the log of the ratio of the 5-year rate to the 3-month rate for the years 1923-1952 was constant at its mean for 1953-2005. While this may be viewed as a strong assumption, two arguments defend it. First, using this procedure to construct an approximation to the 5-year rate for 1953-2005 results in a series that is a good approximation of the actual series. Second, the imputation covers a period of time of fairly low interest rates generally, so it is viewed that any reasonable alternative procedure would have little effect for simulations generating data for 1962 and later.

The constant real rate of interest $r$ is calibrated to be the mean over the years 1923-2005 of the real rate implied by the NMFA rate and the inflation rate series; this gives $r = 2.18\%$.

As discussed above, the appropriate aggregate analogous to the household measure of MA is M2M. Data on the (weighted average) rate of interest paid on M2M is available from the Federal Reserve Bank of St. Louis from 1959. To obtain a longer series, I impute values for earlier periods in a manner analogous to that described above for the 5-year rate series.

The next step is to construct a series for the opportunity cost $\tilde{r}_t$, which is the crucial determinant of behavior in my model. I measure the opportunity cost $\tilde{r}_t$ for

<table>
<thead>
<tr>
<th></th>
<th>mean log($w/y$)</th>
<th>std log($w/y$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No H.S. degree</td>
<td>-3.0021</td>
<td>0.6872</td>
</tr>
<tr>
<td>H.S. Grad</td>
<td>-2.4857</td>
<td>0.9743</td>
</tr>
<tr>
<td>College Grad</td>
<td>-1.6953</td>
<td>1.2552</td>
</tr>
</tbody>
</table>

Table 5: Distribution of households' initial normalized wealth.
Table 6: Properties of the opportunity cost process.

<table>
<thead>
<tr>
<th></th>
<th>(\rho)</th>
<th>((1 - \rho)\mu)</th>
<th>(\sigma_\varepsilon)</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.9714</td>
<td>-.0960</td>
<td>.1064</td>
<td>.9420</td>
</tr>
<tr>
<td></td>
<td>(.0266)</td>
<td>(.0917)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1923-2005 as

\[ \bar{r}_t = \frac{1 + in\text{fl}_t}{1 + MA\text{rate}_t} - \frac{1 + in\text{fl}_t}{1 + NMF\text{Arate}_t}, \]  

where \(in\text{fl}_t\) is the inflation rate, \(MA\text{rate}_t\) is the M2M own-rate, and \(NMF\text{Arate}_t\) is the 5-year T-note rate. I then fit the opportunity cost series to an AR(1) specification as

\[ \ln \bar{r}_t = (1 - \rho)\mu + \rho \ln \bar{r}_t + \varepsilon_t. \]  

These estimates are shown in the table below. The opportunity cost series is shown in Figure 6. In solving the model, I implement this AR(1) approximation of the opportunity cost process and back out the implied real MA cost \(\pi_t\) faced by the households from this process. The unconditional mean of the implied opportunity cost process is 3.85%, implying a mean for \(\pi_t\) of 0.82%.\(^{14}\) Details of the solution and the simulation are given in an Appendix.

[These numbers need to be updated!!!]

5.3 Other Calibrated Parameters

In the quantitative analysis to follow, I set the elasticity parameter in the shopping technology \(\mu_2\) equal to 1 throughout. In this case, NMFA market participants can be expected to exhibit behavior close to that implied by the Baumol-tobin model, as shown above. Experimenting with estimation of this parameter within the empirical framework described below, it was found that this parameter is not well-identified. Moreover, setting \(\mu_2 = 1\) affords the convenience of a quadratic expression for the optimal ratio of consumption to MA balances for NMFA market participants (c.f., equation (26)). As a result, the computational burden of the estimation procedure described below is greatly relieved by this assumption.

I also take the discount factor to be equal to .95 throughout the analysis. This should be a relatively uncontroversial choice for calibration. Setting \(\beta\) a priori reduces the already considerable computational burden of estimation of the remaining parameters.

\(^{14}\)The unconditional distn of \(\ln \bar{r}_t\) is normal with mean \(\mu = -3.6205\) and std dev \(\sigma_\varepsilon = \sigma_\varepsilon/\sqrt{1 - \rho^2} = .4793\). Thus, the unconditional mean of \(\bar{r}_t\) is \(\exp (\mu + \frac{1}{2}\sigma_\varepsilon^2) = 0.0300\).
Figure 9: M2M Own-rate, 5-Year T-Note rate, and implied M2M opportunity cost. The M2M Own-rate is imputed for years before 1959, and the T-note rate is imputed for years before 1953.
5.4 Estimated Parameters

In the next section, I report the results of experiments based on three specifications of the model. The first imposes that all fixed costs are equal to zero; this is the no-fixed costs (NFC) specification. The second allows only for fixed costs of the start-up variety; I call this the start-up costs (SC) model. The third specification allows only for fixed costs of the per-period variety; accordingly, I refer to this one as the per-period costs (PC) model.

In order to conduct these quantitative experiments for one of these specifications, the remaining preliminary task is to set the values of the shopping technology parameter \( \mu_1 (Z^i_t) \), and the fixed cost parameters \( x_0 (Z^i_t) \) or \( x_1 (Z^i_t) \). To do so, I estimate the parameters by minimizing a quadratic approximation of a simulated method of moments criterion function. This procedure is described briefly in the following paragraphs; more detail is provided in the Appendix.

Using the SCF data with the imputations described, I construct profiles of mean log-MA balances, participation rates, and mean log-total financial wealth across age groups 26 to 65 within each education group. These objects are compared to the analogous ones constructed by simulating the model. More precisely, for a given parameterization, I simulate the behavior of 100 households in each age group for each birth-year from 1923 to 1989 conditioning on the realized aggregate opportunity cost process, and extract the relevant profiles for the years 1962, 1983, and 1989. The distance between the data and the simulated profiles is evaluated through the lens of the quadratic form with weighting matrix taken to be the inverse of the estimated covariance matrix from the data moments.

Bowing to computational expense, however, I depart from standard MSM procedure as follows. First, I obtain the criterion function values by the procedure described in the previous paragraph on a sparse grid of values for the parameters to be estimated. Then within each education group, I construct a log-quadratic approximation of the relationship between the two parameters \( \mu_1 (Z^i_t) \) and \( x_0 (Z^i_t) \) (for the SC specification, say) and the criterion, and I choose the parameters used in the quantitative studies as the ones that minimize this approximate fit criterion. (Separation by education group is possible, because the effect on the criterion function of one groups parameters is invariant to the parameters relevant to the other groups.)
Model Parameter Estimates

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu_1$</td>
</tr>
<tr>
<td>NFC</td>
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</tr>
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<td>ND</td>
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</tr>
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<td>HS</td>
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</tr>
<tr>
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<tr>
<td>HS</td>
<td>.002</td>
</tr>
<tr>
<td>Coll</td>
<td>.0026</td>
</tr>
</tbody>
</table>

Table 7: Estimated parameters (Preliminary).

6 Results

6.1 The Fit of the Model to the Data

Table 10 reports the values of the estimated parameters for the three models considered and for each education group within each model. These are the values that are applied in the numerical exercises to follow. It is important to acknowledge that these estimates represent incomplete convergence of the estimation routine. Thus, I have not reported sensitivity measures in the table. The “fit” value in each row is the contribution to the MSM criterion function of the moments relevant to the relevant subsample. Given that I have allowed both of the estimated parameters to vary across education groups, these contributions may be evaluated independently.

There are three things that should be noticed from the table. First, comparing the NFC model to the models that allow for fixed costs, it appears that the ability of the model to explain the behavior of less educated households is improved by allowing for fixed costs. Second, there is little evidence that college educated households face positive fixed costs. Third and relatedly, estimates of fixed costs decrease with education; this result is quite robust and is certain to be maintained by more precise estimates.\(^\text{15}\)

\(^{15}\)Corroborated by other preliminary experiments the results reported in this paragraph appear to be quite robust. On the other hand, although I believe the estimated technology parameters $\mu_1$
Figures 10-12 show (for the NFC, SC, and PC specifications, respectively) simulated age-profiles of log-MA, participation rates, and log-total financial wealth for the three education groups superimposed upon analogous statistics in the data. Each panel shows the model behavior and data for the years 1962, 1983, and 1989. The model profiles represent the mean of the optimal choices of a population consisting of 100 simulated households per cohort. The data profiles were smoothed by taking moving averages within 5 age cohorts.

Comparing across the different specifications, it is apparent where the no-fixed-costs model is deficient: fixed costs are apparently necessary to explain why households in the no-degree and the high-school-degree groups participate in low numbers. It is also apparent why the estimation assigns a higher value of the shopping technology parameter in the NFC specification. In particular, the high level of the profiles of are reasonably accurate, I am not confident to make statements about the ordering across groups or specifications at this time.

Figure 10: Simulated data profiles for the no-fixed-costs (NFC) specification.
Figure 11: Simulated data profiles for the start-up costs (SC) specification.
Figure 12: Simulated data profiles for the per-period (PC) specification.
MA balances in the simulation of the NFC model (top row in Figure 10) is indicative that the estimation attempts to compensate for the high value of participation by inducing a greater need of MA in making transactions.

Having viewed these shortcomings of the NFC model, I will restrict attention in the rest of the analysis to specifications with fixed costs.

There are several more fundamental features of the simulations (and the data) that should be noted. First, financial wealth increases before retirement, and households dissave after retirement to supplement reduced income. It is apparent, but not uncommon in standard life-cycle models, that the model does not reproduce behavior after retirement very accurately.\(^{16}\)

Similarly, participation increases with age; in particular, participation accelerates greatly after ages 30 and 40 (respectively) for college and high-school educated households, and increases somewhat after age 50 for households with less education. This feature of the model suggests that, consistent with findings of Gourichas-Parker, households require only a “buffer-stock” of wealth early in life, and begin to accumulate more wealth later in life in preparation for retirement.

Next, observe that the model generally shows too little participation early in life and too much around the retirement years. This is probably explained by the presence of more heterogeneity in participation costs in the data than in the model. Age-dependence of the participation cost may also be a factor.

There is a significant shift of participation behavior identifiable between 1962 and 1983. This reflected in the profile of MA demand (and total financial assets to a lesser extent). This finding is quite consistent with the data presented in Section 2.

Finally, note the greater steepness of the profile of total assets of more highly educated households. This reflects three influences. First, these households have the steepest income profile among the education groups. Second, they have the lowest replacement rates. Third, participation affords access to higher yielding assets, which encourages more saving.

Figures 13 and 14 show the implications of the SC and PC models, respectively, for the relationship between the construct from the model analogous to the measure of inverse velocity shown in Figure 2 for the aggregate data. In Figure 13, we see evidence of the shift of the response to changes in the opportunity costs of aggregates under the start-up costs model. This effect is due to the hysteresis of behavior induced as a larger fraction of the population becomes experienced. In contrast, Figure 14 shows that the shift is mostly absent in the simulation of the per-period

\(^{16}\)This could be rectified to some (possibly great) degree simply by adjusting the degree of uncertainty households face after retirement. I have not systematically conducted such experiments.
Figure 13: M2M opportunity cost and the ratio of aggregate MA to aggregate labor income for the simulated population in the start-up costs (SC) model.
Figure 14: M2M opportunity cost and the ratio of aggregate MA to aggregate labor income for the simulated population in the per-period fixed costs (PC) model.
Figure 15: MA velocity in the SC model compared to 1.6 times GDP-M2M velocity in the data. The MA opportunity cost is superimposed for reference.

Velocity in the model, calculated as the ratio of aggregated labor income divided by aggregate MA balances, is compared to velocity computed from the aggregate data in Figure 15 for the start-up costs model. Allowing for the scaling factor, which stems primarily from the fact that only households are included in the measure of velocity in the model, it is clear that the model does a good job of replicating aggregate movements of velocity.

The best-fit constant elasticity curves for the SC model plot exhibit elasticities $-0.24$ for the 1959-69 period, and $-0.42$ for the 1977-1999 period; those for the PC model show elasticities $-0.46$ and $-0.56$, respectively.

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$^{17}$The best-fit constant elasticity curves for the SC model plot exhibit elasticities $-0.24$ for the 1959-69 period, and $-0.42$ for the 1977-1999 period; those for the PC model show elasticities $-0.46$ and $-0.56$, respectively.
Figure 16: Mean opportunity cost elasticity of households’ MA balances by age, education and year. Left column shows SC specification; right column shows PC specification.

6.2 The Opportunity Cost Elasticity of Households’ MA Balances

For the calibrated parameterizations, I compute numerically the elasticity of each simulated household’s MA holdings with respect to the opportunity cost. The average elasticity within year, age, and education groups are shown in Figure 16 below; these calculations do not account for the extensive margin.

Several points should be noted. First, these average elasticities are primarily reflective of the degree of participation within the group. These average elasticities rise toward retirement, because households increase their savings as retirement nears. Thus, these households cross their transactions thresholds, at a minimum. They are also more likely to participate in the NMFA markets. Moreover, since the participation cost need only be paid once under the start-up cost model, older households are more likely to be experienced. Once the fixed cost has been paid, most households
sustain their participation.

Next, consistent with their having lower costs of participation, and consequently participating in greater numbers, more educated households MA demand is more elastic than that of households with less education.

Third, across time, the years during and after episodes of high opportunity costs show higher average elasticities. This reflects higher value of participation and higher actual participation as a consequence. Since the start-up cost needs only to be paid once, there is also hysteresis in participation, so that households tend to continue to participate in 1989 (for example) if they were induced to do so in the early 1980s when the value of doing so was much higher.

Figure 17 depicts the relationship between the responsiveness of household MA balances and household wealth in the start-up cost model, and the change in that relationship over time. More precisely, the figure shows mean elasticity within centiles
of the wealth distribution plotted against the mean within the centile for the years 1962, 1983 and 1989 for the simulated population; again, the contribution of the extensive margin is neglected here. Two things are noteworthy. First, reflecting the presence of fixed costs (and to a lesser extent, liquidity constraints), households with lower levels of wealth exhibit less elasticity in this response. Second, induced by the rise of the opportunity cost, very significant changes are evident across years.

Figure 18 shows the mean elasticity exhibited by NMFA market participants and non-participants. Note first the invariance of the elasticity of NMFA market participants. This feature suggest that, conditional on participation, households’ behavior may be well-described by the inventory theoretic model. The mean elasticity of non-participants, however, fluctuates considerably, from a low of (absolute value) 0.0753 at the beginning of the episode reaching a high of 0.2218 in 1981.

Figure 18: Mean elasticity of MA balances of households conditional on participation or non-participation under the start-up costs model.
6.3 Aggregate MA Elasticity and its Decomposition

In this subsection, I report the elasticity of aggregate MA balances as implied by the start-up costs model for the period under study. To compute the elasticity, I construct each of the components described by the decomposition in Subsection 4.3 numerically. For the intensive margin effects, these objects are simply expectations with respect to the endogenous distribution, say \( \omega_{a,t} (H, P, \xi, Z) \), of household characteristics; as such they can be approximated accurately and straightforwardly by averaging over the simulated population.\(^{18}\) The calculation of the terms

\[
\bar{\omega}_t \equiv \sum_{a=1}^{N} \sum_{\xi \in \{0,1\}} \sum_{Z \in Z} \int_P \omega_{a,t} (\bar{H}_a (P, \xi, \pi_t, Z), P, \xi, Z) \, dP
\]

and integration with respect to the marginal density \( \tilde{\omega}_{a,t} (P, \xi, Z) \) to compute the other components of the extensive margin effect is more problematic, since it involves evaluating the endogenous density function at points that may be remote from most of the mass of the distribution.

One way to proceed would be to assume that the marginal distribution of \( H \) and \( P \) is bivariate lognormal, and to estimate the parameters of this distribution with cells of the simulated data defined by \((\xi, Z, a, t)\). Unfortunately, some of these cells have scant or no observations even under very large simulations. To persevere, I assume that the distribution \( \omega_{a,t} \) can be decomposed into components as

\[
\omega_{a,t} (H, P, \xi, Z) = \omega_a^0 (H, P|\xi, Z) \text{Prob}_{a,t} (\xi, Z),
\]

where \( \omega_a^0 (H, P|\xi, Z) \) is the distribution of \((H, P)\) conditional on \((\xi, Z)\), which is assumed to be lognormal and independent of \( t \); and \( \text{Prob}_{a,t} (\xi, Z) \) is the probability at a household drawn randomly at \( t \) has characteristics \((\xi, Z, a)\). The significant assumption is that the effect of time on the distribution of \((H, P)\) comes through changes of the distribution of households’ NMFA market experience. Under this simplifying assumption, it is a straightforward exercise to estimate the parameters of the joint lognormal distribution \( \omega_a^0 (H, P|\xi, Z) \) within subpopulations defined by \((\xi, Z, a)\); and the computation of the \( \tilde{\omega}_t \), and the integration with respect to the density

\[
\tilde{\omega}_{a,t} (P, \xi, Z) \equiv \frac{\omega_{a,t} (\bar{H}_a (P, \xi, \pi_t), P, \xi, Z)}{\omega_t}
\]

\(^{18}\)Note that I have made explicit the dependence upon idiosyncratic characteristics \( Z \) (i.e., education) that was suppressed in previous discussion.
can be performed using standard numerical integration techniques.

Figure 19 shows the aggregate elasticity of MA balances under the start-up costs model, and its decomposition into three broad constituent parts. Although there is an apparent hump in magnitudes of the contributions from participants and non-participants on the intensive margin, it is also clear that the dramatic rise and fall of aggregate elasticity has been driven by the contribution of the extensive margin. While the extensive margin never accounts for more than half of the aggregate elasticity, it induces most of the change of the elasticity over the period.

The times series of each of the determinants of the extensive margin effect are shown in Figure 20. Panel (1) shows the mean of the density of households at the participation threshold \( \bar{\omega}_t \). Consistent with the intuition offered in Subsection 4.3, this object varies greatly over time, closely mimicking the time-series of the opportunity cost. The expectations in Panels (2) and (3) corroborate this view.
Figure 20: The figure shows the components of the effect of the extensive margin on aggregate MA elasticity. Panel (1) shows \( \tilde{\omega}_t \); panel (2) shows \( \tilde{\omega}_t \tilde{E}_{\theta,t} [\tilde{\varepsilon}_a (\xi, \pi_t) H_a (P, \xi, \pi_t)] \); panel (3) shows \( \tilde{E}_{\theta,t} [\Delta_a (H_a (P, \xi, \pi_t), \xi, \pi_t) / M_t] \); and panel (4) shows the product of \( \tilde{\omega}_t \) and the covariance of \( \tilde{\varepsilon}_a (\xi, \pi_t) H_a (P, \xi, \pi_t) \) and \( \Delta_a (H_a (P, \xi, \pi_t), \xi, \pi_t) / M_t \).
Perhaps the most intuitive evidence for this view comes from Panel (2), which shows the product of $\omega_t$ and the expected change of the participation threshold. In sum, these results ascribe a great deal of importance for the determination of the elasticity of aggregate MA to the location of the participation threshold relative to the mass of the current resources of the population.

7 Conclusions

I have described the properties of a class of portfolio choice models with emphasis on understanding the implications for households demand for monetary assets (MA). The model augments a standard life-cycle model with random income fluctuation and borrowing constraints in two ways. First, I incorporate a “shopping technology” necessitating the use of MA in making transactions. Second, I allow for the possibility that payment of a fixed cost may be necessary to access the non-monetary financial assets (NMFA) market.

With respect to capturing the behavior in the data, I find that positive fixed costs are necessary to do so. The cross-sectional data that I use does not offer strong evidence to discriminate between the model with a one time start-up cost, and the one with a per-period cost. Comparing more informally the aggregate series induced by each model, however, shows that the start-up costs model better captures the “shift” that seems to be evident in the data. From both of these models, there is strong evidence that less educated households face higher fixed costs.

Conditional on participation, households MA balances under the model exhibit a constant elasticity to the interest close to (absolute value) .50; thus, their behavior may be well-described by the conventional inventory theoretic analysis. The money demand of households that do not participate in the NMFA market exhibits a small but moderately time-varying response to changes in the opportunity cost; the calculation here suggests the mean elasticity of non-participants fluctuated between (absolute value) .07 and .22 between 1962 and 2005.

The opportunity cost elasticity of aggregated MA balances varies dramatically over the period, rising from a low of .15 in the early 1960s to a peak of .62 in 1981, before settling back to .22 in 2005, the last year of the simulation. The aggregate elasticity is decomposed as the sum of contributions from the intensive margin of participants and non-participants, and an extensive margin. While the intensive margin effects vary moderately over the sample period, most of the variability in aggregate MA elasticity stems from large fluctuations of the extensive margin contribution. This contribution varies from almost zero to .29 over the period.
Looking more closely at the extensive margin reveals an explanation for the varying contribution to aggregate MA elasticity within the context of the theory. I show that an important determinant of the strength of this effect is the density of households’ financial resources evaluated at the threshold level of financial resources sufficient to induce participation. The parameterization of the model examined here implies that the threshold was irrelevant to most households at levels of the opportunity cost extant before the late 1960s; thereafter, however, rising opportunity costs caused a reduction of the threshold so that it became relevant to the decisions of a significant fraction of the population. I suggest that this effect is an important piece of the explanation for the “instability” in aggregate money demand relationships.

References


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