Predictability and Power in Legislative Bargaining

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Abstract

This paper examines the effect of the predictability of recognition processes on the concentration of political power in legislative bargaining. For a broad class of legislative bargaining games, we identify a mild predictability condition on the recognition rule, requiring an ability to rule out some minimum number of legislators as the next proposer, under which Markovian equilibria deliver all economic surplus to the first proposer. That result holds irrespective of whether legislators are patient or impatient. When legislators can be nearly certain that the next proposer belongs to a class of the requisite size, the first proposer receives nearly all of the surplus.
1 Introduction

Legislative bargaining is a nearly ubiquitous feature of public policy-making. The most widely studied formal model of such processes is due to Baron and Ferejohn (1989). In the closed-rule version of their framework, one of \( n \) legislators is randomly selected to make a proposal concerning the allocation of a fixed payoff. If the legislature accepts the proposal, the game ends. If it rejects the proposal, the process repeats (possibly forever). All legislators discount future payoffs at the rate \( \delta \). The Baron-Ferejohn model has served as a workhorse for studying various aspects of legislative bargaining. The properties of the original model have been thoroughly examined, and the framework has been enriched in a number of ways.\(^1\)

One important and potentially unrealistic feature of the Baron-Ferejohn framework is that all uncertainty concerning the identity of the proposer in a particular round is resolved immediately before the proposal is made. Thus, as round \( t \) approaches, legislators do not update the probability with which any given member will be selected as the round \( t \) proposer. Without this assumption, the model becomes non-stationary and (it would seem) less tractable. On substantive grounds, the assumption can be defended as a stylized attempt to capture the unpredictable nature of legislative politics. And yet the processes by which legislatures recognize members to make proposals may be considerably more predictable than the assumption implies, at least in the short-term, for three reasons.

First, even if the recognition process is random, proposers may be pre-announced. For example, a chair may be inscrutable (and hence apparently random), but may nevertheless always identify the next proposer when recognizing the current one (e.g., “Now we’ll hear from Mr. X, and then I have Ms. Y”). In that particular example, all uncertainty regarding the next proposer is resolved exactly one period in advance.

Second, depending on the rules of the legislature, certain candidates for the proposer may be ruled out in advance. For example, the rules may preclude either an individual or members of the same party from being recognized twice in a row. Alternatively, the proposer for round $t$ may be selected in that round from a slate of nominees, who are announced in round $t - 1$ (or earlier).

Third, the selection of the round $t$ proposer may depend upon strategic choices that are themselves predictable in equilibrium. For instance, the choice of the proposer may be up to a chair who is elected in advance, and who has a known “pecking order” of favorites. Even with uncertainty concerning the chair’s favorites, legislators would update their probabilistic assessments concerning the identity of the round $t$ proposer once the presiding chair for round $t$ is elected.

In this paper, we modify the Baron-Ferejohn framework to allow in a general way for a limited degree of predictability in the proposer recognition process. Our model subsumes a wide variety of possibilities, including all of the examples discussed above, as well as many others. Recall that in the stationary equilibria of the Baron-Ferejohn model, the fraction of surplus received by the first proposer can be as low as one-half, and depends on the legislators’ discounts factors.\(^2\) In contrast, our central result is that, under a relatively weak predictability condition, the first proposer receives the entire prize (in the analog of stationary equilibria), irrespective of $\delta$. When a proposal must receive $k$ (of $n$) votes to pass, the predictability condition states that, one period in advance, the legislators can rule out at least $k + 1$ members as the next proposer. Moreover, when this condition “almost” holds (i.e., the legislators can rule out at least $k + 1$ members as the next proposer with near certainty), the first proposer receives nearly the entire prize.

We do not mean to offer our result as a literal description of legislative outcomes. Rather,
we view our messages as “directional”: first proposers likely have considerably more power than the logic of the Baron-Ferejohn framework suggests; that power may be far less sensitive to the patience of the legislators (or equivalently the frequency of legislative deliberations) than has been assumed; and, perhaps most importantly, the predictability (or inscrutability) of a legislature’s recognition process may be a key factor in determining the allocation of rents from public policies.\(^3\)

Our analysis bears some relation to Baron and Ferejohn’s (1989) analysis of recognition processes with asymmetric recognition probabilities. In particular, they showed that, if the recognition probability for legislator \(i\), \(p_i\), is sufficiently high, then \(i\) will receive the entire prize. Clearly, when \(p_i\) is close to unity, the recognition process is predictable, in the sense that all legislators can predict that \(i\) is virtually certain to be chosen as the proposer in the next period (as well as in all future periods). But such circumstances transparently establish \(i\) as a (near) dictator. Our main result differs from the Baron-Ferejohn point because we demonstrate that the first proposer receives the entire prize even if (i) his probability of being selected in any future period is low, (ii) it is predictable that someone else (or some other group of individuals) will make the next proposal, and (iii) subsequent proposers are not at all predictable.

Our analysis also helps explain why different models of legislative bargaining have yielded different conclusions about the concentration of political power. In particular, Bernheim, Rangel, and Rayo (2006) established that, in a model with real-time agenda setting and evolving status quos, the last proposer is effectively a dictator. Their analysis, like ours, presupposed a limited degree of predictability in the recognition process. Thus, in both settings predictability yields highly concentrated political power; the details of the bargaining process determine where that power resides.

The paper proceeds as follows. Section 2 conveys the intuition for our results through

\(^3\)Alternatively, one can read our analysis as a methodological criticism of the Baron-Ferejohn framework – i.e., as showing that reasonable modifications lead to unreasonably extreme implications. We do not, however, favor that interpretation.
some simple examples. We present our framework in Section 3 and our main results in Sections 4 and 5. We consider an extension with weighted voting in Section 6. We discuss interpretations and applications in Section 7. Section 8 concludes.

2 Some Simple Examples

In this section, we both motivate the directions in which we generalize the Baron-Ferejohn framework, and provide intuition for our central results, through a series of relatively simple examples. Each example takes as its starting point the basic Baron-Ferejohn closed-rule infinite-horizon model, in which all legislators are recognized with equal probability, and legislative approval requires a majority.

Example 1: Modify the basic model so that uncertainty concerning the round $t$ proposer is resolved and revealed in round $t - 1$, rather than in round $t$.

It is easy to construct a Markov-perfect equilibrium in which the first proposer receives the entire prize. The strategies are simple: in every period, the selected legislator proposes to keep the prize for himself, and all legislators (except for the next proposer, who is known) vote in favor. Given the continuation equilibrium, voting against the proposal would simply shift the prize from the current proposer to the next proposer, which is of no benefit to other legislators. Thus, we plainly have an equilibrium. While the equilibrium might seem to rely on the fortuitous resolution of indifference among voters, this is not the case: if the proposer offered a tiny share to a set of legislators just large enough to assure passage, they would all have strict incentives to vote in favor.

Establishing uniqueness of the Markov-perfect payoffs is somewhat more involved. The argument is entirely transparent in a finite-horizon version of the model with zero discounting, where the default payoff to all legislators (if no proposal is accepted) is zero; it is a direct consequence of backward recursion. In particular, the last proposer would clearly receive the entire prize (if necessary offering vanishingly small payments to a subset of leg-
islators to assure passage). Knowing this, and knowing the identity of the last proposer, the second-to-last proposer can obtain the entire prize, and so forth. Now suppose instead that the default policy is some division of the prize. Then the last proposer can clearly claim at least half the prize (by offering vanishingly small payments to the bare minority of colleagues who receive the smallest shares in the default allocation). It follows that the second-to-last proposer, knowing who the last proposer will be, can identify a bare minority who collectively expect to receive no more than one-quarter of the prize; hence, by offering members of that group vanishingly small payments, he can claim at least three-quarters of the prize. By the same logic, the third-to-last proposer can claim seven-eighths of the prize, the fourth-to-last fifteen-sixteenths of the prize, and so forth. Thus, the first proposer can claim nearly all of the prize. In the infinite horizon model (for Markov-perfect equilibria), one can, in effect, perform this recursion starting from any future period (since the argument does not depend on which outcome would result if that period were reached). Accordingly, we can pass to the limit and conclude that the first proposer receives the entire prize.

**Example 2:** Modify the basic model by assuming that the round $t$ proposer is randomly selected (with equal probabilities) in round $t$ from a set of $n^*$ nominees, call it $N_t^*$. The nominees are in turn selected randomly (with equal probabilities) in round $t - 1$ from the full set of legislators, and immediately announced. We will suppose that the list of nominees is not too long: $n^* < \frac{n}{2} - 1$.

Analogously to Example 1, it is easy to see that the following is an equilibrium: in round $t$, the selected legislator always proposes to keep the entire prize for himself, and the proposal passes with the support of legislators belonging to $N \setminus N_{t+1}^*$ (where we again implicitly resolve their indifference by assuming that the proposer offers them vanishingly small shares). Members of that group are willing to vote for the proposal because they understand that (according to the continuation equilibrium) rejecting it would simply shift the entire prize from the current proposer to some member of $N_{t+1}^*$. Because $N_t^*$ has fewer than $\frac{n}{2}$ members, $N \setminus N_{t+1}^*$ has more than $\frac{n}{2}$ members; therefore the proposal passes.
The intuition for uniqueness is similar to that given for Example 1. For the finite-horizon version of the model where the default is some division of the prize, the recursive argument is only slightly more subtle. In particular, knowing that the last proposer will receive at least half of the prize allows the second-to-last proposer to conclude that members of \( N \setminus N_{t+1}^* \) will collectively receive less than half; consequently, he can find \( \int \left( \frac{n}{2} + 1 \right) \) members of that set who collectively expect to receive less than \( \frac{\int (\frac{n}{2} + 1)}{n - n^*} \) of the prize. Accordingly, if \( n^* < \frac{n}{2} - 1 \), the second-to-last proposer leaves a smaller fraction of the prize “on the table” than the last proposer. Moving to successively earlier proposers, that fraction shrinks by the same factor of proportionality, converging to zero for the first proposer when the number of periods is large.

**Example 3:** Modify the basic model by assuming that every legislator belongs to one of \( P \) parties, where \( P \geq 3 \). Assume that, despite their party affiliations, each legislator is still concerned with benefiting his own constituents; hence politics remains a zero-sum game. Let \( n_j \) (with \( \sum_{j=1}^{P} n_j = n \)) denote the number of legislators belonging to party \( j \). By convention, list the parties so that \( n_1 > n_2 > ... > n_P \). Also assume \( n_1 < \frac{n}{2} - 1 \), so that no party has a majority. Now consider a recognition rule that cycles through the parties, starting with the largest. For example, a randomly selected member of the largest party makes the first offer, followed by a randomly selected member of the second largest party, and so forth, returning to the largest party if a proposal by the smallest party is rejected. In this setting, the first proposer from the largest party will receive the entire prize in any Markov-perfect equilibrium. The logic is essentially the same as for Example 2.

**Example 4:** Modify the basic model by assuming that the legislature represents \( n \) districts, and that each district has two representatives who share the same preferences (i.e., the policy describes the distribution of benefits across districts). A chair for round \( t \) is selected at random from the set of legislators and announced in round \( t - 1 \). The round \( t \) chair selects the proposer for round \( t \), but cannot choose himself.

Plainly, each chair will recognize the other legislature from his own district. Thus, this
example is essentially equivalent to Example 1, wherein the proposer rather than the chair is announced one round in advance. What is important here, however, is not that each chair can recognize an alter ego, but rather simply that the preferences of each chair are known with sufficient precision to render his choice of proposer sufficiently predictable. Plainly, regardless of the chair’s preferences, any pure strategy equilibrium will have that feature.

To encompass these and other possibilities, our analysis must allow for (i) the resolution and revelation of uncertainty bearing on the selection of the round $t$ proposer prior to round $t$ (as in Examples 1 and 2), (ii) rules that constrain the selection of the round $t$ proposer, based on events prior to round $t$ (as in Example 3), and (iii) actions taken by legislators in round $t$ that directly influence the identity of the round $t$ proposer (as in Example 4). The model presented in the next section is therefore formulated with considerable generality.

3 The Model

We consider a legislature tasked with determining the distribution of a fixed payoff, normalized to unity, among $n$ legislators, the collection of whom we denote $N = \{1, \ldots, n\}$. Thus, the set of possible policies is $X \equiv \{x \in \mathbb{R}^n : 0 \leq x_i \leq 1, \forall i \text{ and } \sum_{i \in N} x_i \leq 1\}$. We use $e_i \in X$ to denote the policy that allocates all surplus to legislator $i$. Collective deliberation entails a potentially infinite sequence of rounds, $t = 0, 1, 2, \ldots \infty$. In each round, one legislator is empowered to propose a policy. If the legislature approves the proposal according to the voting rules described below, the game ends and the policy is implemented. If the legislature fails to approve the proposal, play proceeds to the next round.

Within each round, events unfold as follows:\(^4\)

1. Legislators engage in political maneuvering to influence the selection of current and

\(^4\)If we reverse the ordering of the first two steps, or if we allow information to be revealed both before and after political maneuvering, we will still obtain our main result - that with first-period ($\epsilon$-)predictability the first proposer seizes in equilibrium (almost) the entire share. However, to obtain the result with first-period conditional ($\epsilon$-)predictability and pure strategy equilibria, it is important that no additional information is revealed between a proposal and the next round of maneuvering.
future proposers.

2. Information concerning random events that influence the selection of current and future proposers is revealed.

3. The proposer, \( p^t \in N \), is determined according to a recognition rule.

4. The current proposer makes a proposal.

5. Legislators vote on the proposal.

We model political maneuvering to influence the selection of current and future proposers as the choice, by each legislator \( i \) in each round \( t \), of an action variable \( g^t_i \in G \). Thus, period \( t \) political maneuvering is described by a vector \( g^t = (g^t_1, ..., g^t_n) \). For simplicity, we assume that \( g^t \) is observable. Such maneuvering may include, for example, voting on a future chair and the choice of a proposer by a current chair.

We model the random events that influence the selection of proposers as a sequence of independent random variables, \( \theta^t \in \Theta \), each of which is distributed according to the same probability measure, \( \mu \). At the start of round \( t \), information is potentially revealed concerning \( \theta^t, \theta^{t+1}, ..., \theta^{t+T} \) for some fixed \( T \).

In particular, for \( k = 0, ..., T \), we let \( \mathcal{I}_k \), a partition of \( \Theta \), represent all the possible states of knowledge concerning \( \theta^{t+k} \) as of round \( t \). Thus, in step 1 of round \( t \), the legislators learn, for each \( k = 1, ..., T \), the element of \( \mathcal{I}_k \), denoted \( I_k \), in which \( \theta^{t+k} \) lies. Naturally, for all \( k = 0, ..., T - 1 \), \( \mathcal{I}_k \) must be a weakly finer partition than \( \mathcal{I}_{k+1} \) (so that no information is lost concerning \( \theta^t \) as \( t \) approaches).

Thus, after step 1 of round \( t \), information concerning \( (\theta_t, ..., \theta_{t+T}) \) is summarized by a vector of information sets, \( I^t = (I^t_0, ..., I^t_T) \in \mathcal{I}_1 \times ... \times \mathcal{I}_k \). In addition, prior to round \( t - T \), the

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5 We can also accommodate the case \( T = \infty \).
6 At the cost of some additional notational complexity, one could allow both \( \Theta \) and the information partitions to depend on \( t \).
7 That is, for any \( \theta \in \Theta \), let \( I_k \) and \( I_{k+1} \) be the elements of \( \mathcal{I}_k \) and \( \mathcal{I}_{k+1} \) that contain \( \theta \), respectively. Then \( I_k \subseteq I_{k+1} \).
legislators know only that $\theta^t \in \Theta$. Finally, we assume that $\theta^t$ is fully revealed when $p^t$ is announced in round $t$ (e.g., because the recognition rule is invertible within each $I_0 \in I_0$).\footnote{One can relax this assumption at the cost of some additional notation.}

We allow the recognition rule applied in each round $t$ to depend on the identities of past proposers (e.g., the rule might prevent a legislator from proposing twice in a row), past and current random events, and past and current political maneuvering. Let $h^t_P = (p^0, p^1, ..., p^{t-1})$, $h^t_\theta = (\theta^0, \theta^1, ..., \theta^{t-1})$, and $h^t_G = (g^0, g^1, ..., g^{t-1})$ denote the histories of, respectively, proposers, random shocks, and politicking up to (but not including) period $t$. The recognition rule is some deterministic function $P^t(h^t_P, h^t_\theta, h^t_G)$, that assigns a round $t$ proposer, $p^t$.

Once the proposer $p^t$ makes a proposal, $x^t \in X$, in step 4 of round $t$, other legislators vote on the proposal in some fixed sequential order. A proposal is implemented if and only if at least $k$ legislators (other than the proposer) vote in favor, where $k < n - 1$ (i.e., we rule out unanimity rule).

If the policy $x$ is implemented in round $t$, the payoff to legislator $i$ is

$$u_i(x, t, h_{G}^{t+1}) = \delta^t x_i + \sum_{s=0}^{t} \delta^s v(g^t_i)$$

We assume that $v(g_0) = 0$ for some “neutral” action $g_0$, and $v(g_i) \leq 0$ for all $g_i \in G$. Thus, no legislator has an incentive to prolong negotiations due to enjoyment derived from political maneuvering. Note that the discount factor, $\delta$, is the same for all legislators.

Given the generality with which we have treated the recognition rule, the game is potentially non-stationary. Nevertheless, within any round $t$, many subgames are structurally identical. For example, because the recognition rule does not depend on which rejected proposals were made in the past or the vote patterns by which legislators rejected those proposals, differences in histories that are confined to those events do not affect the structure of the resulting subgame. To put the matter more formally, we can define the state of the
game in stage 1 of round $t$ as $s_t^1 = (h_t^P, h_t^\theta, h_t^G, I_t^{t-1})$ \(^9\), the state in stage 4 of round $t$ as $s_t^4 = (h_{t+1}^P, h_{t+1}^\theta, h_{t+1}^G, I_t^t)$, and the state in stage 5 of round $t$ as $s_t^5 = (h_{t+1}^P, h_{t+1}^\theta, h_{t+1}^G, I_t^t, x^t)$\(^10\).

Let $S_t^i$ denote the set of possible states for step $i$ of round $t$. If two subgames starting in stage $i$ of round $t$ are characterized by the same state $s_t^i$, they are structurally identical.

After proving a folk theorem for subgame-perfect equilibria, Baron and Ferejohn restricted attention to stationary equilibria. Because our model is non-stationary, we cannot do the same. Yet if we are to avoid a folk theorem, some similar restriction is required. Baron and Ferejohn’s stationarity requirement is equivalent to the restriction that equilibrium strategies are the same across all structural identical subgames. In models with an evolving state variable, where the state space is identical in all periods, that requirement limits attention to strategies that depend only on the current value of the state variable, i.e., to Markov-perfect equilibrium. In our model, the set of possible states changes from one round to the next (because $S_t^i$ summarizes a history of increasing length). Still, in the spirit of Baron and Ferejohn’s analysis, we refine the set of equilibria by requiring that any equilibrium prescribe the same continuation strategies in all structurally identical subgames. We refer to such equilibria as Markovian. Our focus on Markovian equilibria rules out strategies for which choices depend on past proposals and votes, inasmuch as those actions have no structural implications for the continuation game.

In focusing on Markovian equilibria, we follow the general practice in the literature on legislative bargaining. Substantively, one can justify this focus by appealing to the relative simplicity of such equilibria. In general, one cannot define valid equilibria without allowing strategies to depend on variables that alter structural features of the continuation game. Non-Markovian equilibria are more complicated because strategies also depend on variables that have no structural implications for the continuation game. Inasmuch as non-Markovian equilibria require legislators to follow different continuation strategies in structurally identical circumstances, sustaining any such equilibrium presumably requires a

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\(^9\)We let $I^{t-1}$ be some arbitrary element of $I_1 \times \ldots \times I_k$.

\(^10\)We do not bother to define a state for stages 1 or 3 because no decisions are taken.
higher degree of coordination. Yet because the problem at hand is purely one of distribution, complex coordination is never in the legislators’ mutual interests.

Compared with Baron and Ferejohn (1989), our model of legislative bargaining adopts a closed rule and allows for more general recognition rules. To see that it subsumes Baron and Ferejohn’s closed-rule model, let \( G \) be degenerate, \( \Theta = \{1, \ldots, n\} \), \( F(\theta) = \frac{\theta}{n} \) (i.e., a uniform distribution over \( \Theta \)), \( P^t(h^t_p, h^t_{G}, h^t_{G} + 1) = \theta^t \) (so that every legislator is chosen as the round \( t \) proposer with equal probability), and \( I_k = \{1, \ldots, n\} \) for all \( k \) (so that no information concerning \( \theta^t \) is revealed before \( p^t \) is selected). The following are some examples of simple alternative processes that our framework also subsumes.

(i) The process is as in Baron and Ferejohn, except that \( I_1 = \{1, \{2\}, \ldots, \{n\}\} \), so that the round \( t \) proposer is revealed in round \( t - 1 \), prior to the round \( t - 1 \) proposal.

(ii) The process is as in Baron and Ferejohn, except that the same proposer cannot be recognized again for \( \tau \) rounds after making a proposal.

(iii) The role of proposer rotates through the legislators in a fixed order.

(iv) The round \( t \) proposer is chosen in round \( t' \leq t \) through strategic interaction among the legislatures, such as a nomination process followed by a vote.

(v) At most \( m^* \) nominees for the round \( t \) proposer are determined in round \( t'_t < t \), and the proposer is then chosen from the nominees in round \( t \) through strategic voting.

(vi) The round \( t \) proposer is chosen by a chair, who is periodically elected through some strategic process.

Combinations of these types of processes are also subsumed. For example, the round \( t \) proposer could be chosen by a chair subject to the constraint that the proposer cannot have submitted another proposal within the last three rounds; the chair in turn is periodically elected through a strategic process that is also influenced by random events.
As we have explained, our analysis investigates the effect of the predictability of the recognition process on the concentration of political power. To this end, let $\hat{\Theta}^t_{i} (s^t, g^{t+1})$ denote, for any state $s^t$ and $(t+1)$-period maneuvering $g^{t+1}$, the set of $\theta^{t+1}$ for which $i$ becomes the round $t + 1$ proposer:

$$\hat{\Theta}^t_{i} (s^t, g^{t+1}) = \{ \theta^{t+1} \in I^t_1 \mid P^t(h^t_P, h^t_{g^{t+1}}, h^t_{g^{t+1}}) = i \}$$

Next, let $r^t_i(s^t, g^{t+1})$ denote the probability of realizing such a $\theta^{t+1}$ conditional on the knowledge that $\theta^{t+1} \in I^t_1$ (an element of $s^t$):

$$r^t_i(s^t, g^{t+1}) = \frac{\mu(\hat{\Theta}^t_{i} (s^t, g^{t+1}))}{\mu(I^t_1)}$$

Finally, let $\hat{P}^t_\varepsilon(s^t_4)$ denote the set of legislators who would be selected as the round $t + 1$ proposer with probability greater than $\varepsilon$ given some conceivable pattern of round $t + 1$ maneuvering:

$$\hat{P}^t_\varepsilon(s^t_4) = \{ i \in N \mid r^t_i(s^t_4, g^{t+1}) > \varepsilon \text{ for some } g^{t+1} \in G^n \}$$

Thus, if $i \not\in \hat{P}^t_\varepsilon(s^t_4)$, the chances that $i$ will be selected as the proposer for round $t + 1$ is less than $\varepsilon$, regardless of round $t + 1$ maneuvering. Notice that if $i \not\in \hat{P}^t_\varepsilon(s^t_4)$, then $i$ has been ruled out as the proposer for round $t + 1$.

**Definition 1.** The recognition process exhibits **one-period $\varepsilon$-predictability of degree $m$** if for all $t$ and $s^t_4 \in S^t_4$, we have $|\hat{P}^t_\varepsilon(s^t_4)| \leq m$.

If a recognition process exhibits one-period $\varepsilon$-predictability of degree $m$ for $\varepsilon = 0$, we will simply say that it exhibits **one-period predictability of degree $m$**. Thus, one-period predictability of degree $m$ means that, by step 4 of round $t$, at least $n - m$ legislators have been ruled out as the proposer for round $t + 1$. The Baron-Ferejohn process does not exhibit one-period predictability of degree $m$ for any $m < n$. However, processes (i) and (iii) above both exhibit one-period predictability of degree one, process (ii) exhibits one-period
predictability of degree $n - \tau$, and process (v) exhibits one-period predictability of degree $m^*$. In general, the degree of predictability for a given process depends on the timing of decisions that influence the choice of the round $t$ proposer, institutional constraints on the order of proposers, and the timing of revelation for random events.

Note that one-period predictability of degree $m$ has no implications for two-period predictability (defined in the analogous way). For example, though process (i) exhibits one-period predictability of degree one, it does not exhibit two-period predictability of degree $m$ for any $m < n$. Our results will not require $q$-period predictability of any degree for $q > 1$.

Example 4 in Section 2 suggests that we may also be interested in a notion of predictability that is conditional on rational expectations concerning political maneuvering in the next round. Thus, we also define $\hat{P}(s^t_4, g^{t+1})$ as the set of legislators who have more than an $\varepsilon$ chance of being selected as the round $t + 1$ proposer, conditional on political maneuvering $g^{t+1}$ in round $t + 1$:

$$\hat{P}(s^t_4, g^{t+1}) = \{p \in N \mid r^t_i(s^t_4, g^{t+1}) > \varepsilon\}$$

**Definition 2.** The recognition process exhibits one-period conditional $\varepsilon$-predictability of degree $m$ if for all $t$, $s^t_4 \in S^t_4$, and $g^{t+1} \in G$, we have $|\hat{P}(s^t_4, g^{t+1})| \leq m$.

We will use the phrase one-period conditional predictability of degree $m$ to denote one-period conditional $\varepsilon$-predictability of degree $m$ for $\varepsilon = 0$. We note that one-period conditional predictability of degree $m$ is a weaker requirement than one period predictability of degree $m$.

## 4 Main Results

The above model defines a game in which the strategy profiles of the legislators consist of, for every $t = 0, 1, 2, ..., \infty$ and for every state of the relevant stage of the game $s^t_1 \in S^t_1$, $s^t_4 \in S^t_4$ and $s^t_b \in S^t_b$, political maneuvering in the first stage $\{g^t_i(s^t_1) : S^t_1 \rightarrow G\}_{i \in N}$, proposal making in the fourth stage if recognized, $x^t(s^t_4) : S^t_4 \rightarrow X$, and voting in the fifth stage.
\{v_i^t(s_i^t) : S_i^t \rightarrow \{\text{yes, no}\}\}_{i \in \mathbb{N}}. We characterize the (pure and mixed strategy) Markovian equilibrium of the game.\textsuperscript{11}

**Proposition 1. (Main Result)**

For any \(k < n - 1\) and any \(\delta < 1\), if the recognition process exhibits one-period \(\varepsilon\)-predictability of degree \(m < n - k\), then in any (pure and mixed strategy) Markovian equilibrium, for any \(s_4^0 \in S_4^0\), we have \(y^0(s_4^0) \leq \max\{\varepsilon, \frac{\delta}{1-\delta}\varepsilon\}\), where \(y^0(s_4^0) \equiv \max_{i \neq p_x^0} x_i^0(s_4^0)\) is the largest share the period-0 proposer offers to a legislator other than herself.\textsuperscript{12}

**Proof.** See appendix.

\[\square\]

In words, if the recognition process allows us to identify in every period a group of fewer than \((n - k)\) possible proposers for the next period, where all the other legislators have only a small probability of being recognized, then in equilibrium the first proposer seizes almost the entire share for herself.

This result illustrates the equilibrium outcome in the case in which we may not be completely certain about the identities of the possible next period proposers, but as long as we know for certain members, the probability of them being recognized in the next period is fairly small, and as long as there is a large enough number of such unlikely-to-be-recognized legislators, we are able to establish an (equally small) upper bound on the share the first proposer has to give up to members other than herself. One-period predictability with a small amount noise enables the current proposer to exclude the right number of members, and therefore grants her the power to seize almost the entire share for herself. Note that one important feature of our result is that knowledge about recognition in more than one period from the current period \((q\)-period predictability for any \(q > 1\)) is irrelevant.

\textsuperscript{11}We assume throughout this paper that whenever indifferent, the legislators vote for the proposal. So here mixed strategy is possible only for political maneuvering and proposal-making, but not for voting.

\textsuperscript{12}For the case in which the equilibrium proposal-making involves mixed strategy, Lemma 3 in the Appendix shows that for any two proposals a proposer offers with positive probability in a Markovian equilibrium, the highest shares these two proposals give to a legislator other than the proposer herself must be equal. Therefore \(y^0(s_4^0)\) as defined above can also be used in a mixed-strategy equilibrium.
When $\epsilon = 0$, we have an immediate corollary that yields a striking result:

**Corollary 1. (First proposer gets everything)**

For any $k < n - 1$ and any $\delta < 1$, if the recognition process exhibits one-period predictability of degree $m < n - k$, the only (pure and mixed strategy) Markovian equilibrium outcome is $e_p$. In words, the recognized legislator in round 0 proposes to keep the entire prize, and this offer is accepted.

*Proof.* See appendix.

We illustrate the intuition behind this result with the scenario when all legislators adopt pure strategies. Suppose the "first proposer gets all" result is not true, then at least $(n-k)+1$ legislators have strictly positive continuation values - as a matter of fact not only strictly positive, but its current discounted value must be greater than or equal to the largest share the proposer is willing to give among the legislators in the current minimal winning coalition. Out of these legislators, at least one of them cannot be recognized in the next period, so the fact that she has strictly positive continuation value must be due to the fact that some recognized next-period legislator makes a proposal next period to give her a strictly positive share. Following the same logic as in the previous period, such a proposal is optimal for this next-period proposer only if at least $(n-k)+1$ legislators have strictly positive continuation values (in the next next period) – again, not only strictly positive, but its current discounted value must be greater than or equal to the largest share the current proposer is willing to give among the legislators in the current minimal winning coalition. Note that the identities of these $(n-k)+1$ legislators may well differ from the $(n-k)+1$ legislators from the previous period, but their identities are irrelevant for our analysis. Iterating this argument, as long as the discount factor is strictly less than 1 (i.e., the legislators have strict time preference), at a certain point to sustain this logic, giving someone a strictly positive share, the share has to be so large that it exceeds the upper bound of 1. Hence we have the contradiction we
The above result illuminates the power of one-period predictability. In our model of legislative bargaining, predictability implies excludability, that is, enough knowledge about who can be the next-period proposer enables the current proposer to exclude all that have a positive continuation value from the current winning coalition, and in essence grants the current proposer the power to seize the entire share for herself. The predictability of proposers’ identities more than one period in the future does not matter. One-period predictability confers complete power.

5 Conditional Predictability

As Example 4 in Section 2 illustrates, there are situations in which the recognition process does not exhibit one-period predictability of degree $m$ for any $m < n$ (because different choices of $g_{t+1}$ can yield any proposer), but where the identity of the next proposer is nevertheless predictable (because the choice of $g_{t+1}$ is predictable). Although the propositions in the previous section do not apply to that example, the same result nevertheless holds, provided we focus on pure strategy equilibria (so as to render $g_{t+1}$ predictable in equilibrium). In this section, we provide a result on pure strategy equilibria that covers such examples. Naturally, the result requires a somewhat modified notion of predictability.

In the last section, the condition of one-period ($\epsilon$-)predictability requires the recognition rule to identify the same group of $m$ possible next-period proposers no matter what the next-period political maneuverings may be. A weaker condition, as we defined earlier, is one-period conditional ($\epsilon$-)predictability, which requires the recognition rule to identify for every possible profile of next-period political maneuvering a group of $m$ possible next-period proposers. The following result shows that if we focus on pure-strategy Markovian equilibria, we can restore our main result that the first proposer seizes (almost) the entire share.

\footnote{As a matter of fact, as long as we focus on equilibria in which the choices of $g$ are non-random, the same result holds.}
whenever the recognition process exhibits one-period conditional \((\epsilon-)\)predictability of degree less than \(n - k\).

**Proposition 2.** For any \(k < n - 1\) and any \(\delta < 1\), if the recognition process exhibits one-period conditional \(\epsilon\)-predictability of degree \(m < n - k\), then in any pure strategy Markovian equilibrium, for any \(s_1^0 \in S_1^0\), we have \(y^0(s_1^0) \leq \max\{\epsilon, \frac{k}{1-\delta}\epsilon\}\), where \(y^0(s_1^0) \equiv \max_{i \neq p} x_i^0(s_1^0)\) is the largest share the period-0 proposer offers to a legislator other than herself.

**Proof.** See appendix.

As in Section 4, a corollary on one-period conditional predictability follows naturally:

**Corollary 2.** For any \(k < n - 1\) and any \(\delta < 1\), if the recognition process exhibits one-period conditional predictability of degree \(m < n - k\), the only pure strategy Markovian equilibrium outcome is \(e_{p0}\). In words, the recognized legislator in round 0 proposes to keep the entire prize, and this offer is accepted.

**Proof.** See appendix.

Again we illustrate an application of this result. Suppose the proposer is selected from a set of eligible candidates by a chair, who has a strict preference ordering over proposers, and who is selected in some previous round via a process that involves strategy, randomness, or both. (For simplicity, suppose this chair is not chosen from the set of legislators.)

**Corollary 3.** In the aforementioned institution, for any \(k < n - 1\) and any \(\delta < 1\), the only pure strategy Markovian equilibrium outcome is \(e_{p0}\).

**Proof.** See appendix.
6 Extension: Weighted Voting and Parties

As in Example 3 of Section 2, we modify the basic Baron-Ferejohn model by assuming that every legislator belongs to one of $P$ parties, where $P \geq 3$. Let $n_j$ (with $\sum_{j=1}^{P} n_j = n$) denote the number of legislators belonging to party $j$. By convention, list the parties so that $n_1 > n_2 > \ldots > n_P$. We depart here from our previous example as follows. Instead of assuming each legislator is concerned only with his own constituents, we assume that legislators belonging to the same party have identical preferences. Therefore the problem is one of distributing benefits across parties: the set of possible policies is $X \equiv \{ x \in \mathbb{R}^P : 0 \leq x_j \leq 1, \forall j \text{ and } \sum_{j=1}^{P} x_j \leq 1 \}$.

This model is equivalent to our basic model with $P$ legislators where each legislator has a different voting weight ($\frac{n_j}{n}$ for legislator $j$). Thus, while we interpret the model as pertaining to political parties, it also represents legislatures where legislators have unequal vote shares. Consider a recognition rule that cycles through the parties (in a fixed, known order), starting with the largest. For example, a randomly selected member of the largest party makes the first offer, followed by a randomly selected member of the second largest party, and so forth, returning to the largest party if a proposal by the smallest party is rejected. We adopt simple majority rule, so that a proposal is passed if and only if at least $\text{int}\left(\frac{n}{2}\right)$ legislators (other than the proposer) vote in favor. The following result (which is related to but not a corollary of our previous propositions) shows that if none of the parties is too large, the largest party (which makes the first offer) always gets all of the surplus.

**Proposition 3.** If $n_1 < \frac{n}{2} - 1$, in the aforementioned institution, the only Markovian equilibrium outcome is $e_1$, i.e. the largest party receives the entire surplus.

**Proof.** See appendix.

In words, if we consider the distribution of some surplus across parties and the recognition cycles through the parties, as long as the party sizes are not too large (so that no party has
a majority), the party that proposes first receives the entire surplus.

Two observations follow immediately from this result. First, the same conclusion would not follow in a two-party system. Second, smaller parties are strictly better off in a setting where they get to propose in each period with a lower probability, than they are in a system where they take their turn after the larger parties.

7 Discussion

Several points concerning the generality and interpretation of our results merit discussion.

First, the information structure of our model is such that at the time the legislators are making their period-\(t\) decisions (be it proposal-making or voting), the distribution of the next period recognized legislator is common knowledge. If our crucial assumption of one-period predictability is satisfied, i.e. if the recognition rule is such that it has one-period predictability with support less than \(n - k\), then at each period not only is it common knowledge that fewer than \(n - k\) legislators can be recognized as next-period proposers, but also their identities are common knowledge. However, that the identities of all possible next-period proposers are common knowledge is not necessary for our result. Consider a slightly different information structure: it is common knowledge that in any period, the number of possible next-period proposers is less than \(n - k\), but their identities are not common knowledge. As long as at any period, each legislator knows whether or not she has positive probability of being recognized next period, and if this fact is common knowledge, our result will still hold.

Second, the essence of the the main result can be extended to other settings. Consider for example a legislature where only a subset of legislators can be recognized to make offers, and assume this subset must be included in any winning coalition (i.e., they each have veto power). Then we have the result that the entire share is seized by the group of veto players.

Our result can also be applied to situations other than legislative bargaining. For exam-
ple, it can be applied to decision making through direct democracy. Consider a deliberative body making a decision on the distribution of a certain divisible benefit. The deliberative body is divided into two groups, politicians and citizens. Only the politicians are eligible to be recognized to make proposals concerning the distribution, and all (politicians and citizens) vote on the proposal. Suppose they vote sequentially according to simple majority rule (of course same results will apply for other voting rules), so that if more than half of the deliberative body votes for the proposal, the proposal is adopted and the benefit is distributed according to the proposed distribution; otherwise the legislature moves on to the next period in which another politician is recognized to make another proposal. Again assume all (politicians and citizens) discount the future at some common rate. We can then restate our result in this framework as the follows: if the size of the group of politicians is less than a half of the deliberative body (i.e. if the politicians constitute a minority of the deliberative body), in any Markovian equilibrium the first recognized politician will propose keeping the entire surplus for herself, and this proposal will be accepted in equilibrium. As the assumption that politicians constitute a minority in size is quite reasonable in many direct democracies, our results provide a striking observation concerning a tendency within direct democracies: if the power to make proposals is limited to an elite group, benefits will be distributed extremely unequally, even within that group.

8 Conclusion

In this paper, we have developed and solved a new generalized class of legislative bargaining games. We have shown that if the recognition process is such that the legislators are able to put a relatively weak bound on the number of possible next-period proposers, the only Markovian equilibrium outcome is that the first proposer keeps all benefits for herself. When the recognition process respects the bound, all other characteristics of the recognition process (such as the predictability of who will make proposals more than one round in the future) are
irrelevant, and the only equilibrium outcome is for the first proposer to receive everything. We have also shown that the preceding result is robust with respect to certain perturbations, and we considered several applications and extensions.

References


Appendix

Lemma 1. Every Markovian equilibrium reaches immediate agreement in any subgame starting in stage 5 (voting stage) of round $t$ for all $t$.

Proof. Suppose this is not the case in equilibrium for a subgame starting in stage 5 of round $t$ with state $s^t_5 = (h^{t+1}_p, h^{t+1}_g, h^{t+1}_G, I^t, x^t)$, i.e. the proposal being voted upon in the beginning of this subgame is $x^t$. Consider the subgame starting in stage 4 of round $t$ with state $s^t_4 = (h^{t+1}_p, h^{t+1}_g, h^{t+1}_G, I^t)$. Let $V^{t+1}_i(s^t_4)$ denote the continuation value of legislator $i$ at the start of period $t+1$, \(^{14}\) and let $Q^{t+1}(s^t_4)$ denote the set of the $k$ legislators with the lowest next-period continuation value (excluding the proposer, $p^t$), and consider the following proposal: $x^t_i(s^t_4) = \delta V^{t+1}_i(s^t_4)$ if $i \in Q^{t+1}(s^t_4)$; $x^t_i(s^t_4) = 0$ otherwise. This proposal will be accepted by all members of $Q^{t+1}(s^t_4)$, and the proposer gets $1 - \delta \sum_{i \in Q^{t+1}(s^t_4)} V^{t+1}_i(s^t_4) > \delta V^{t+1}_{p^t}(s^t_4)$, because $\sum_{j \in N} V^{t+1}_j(s^t_4) \leq 1$ and $\delta < 1$. And note that the right hand side of the above inequality is precisely the proposer’s (present discounted) continuation value for next period. Therefore, by deviating to this new proposal the proposer can be strictly better off.

\[\square\]

Lemma 2. Any proposal offered with positive probability in a Markovian equilibrium never offers strictly positive shares to more than $k$ legislators.

Proof. Suppose to the contrary, that in equilibrium some proposer offers strictly positive shares to more than $k$ legislators. From the above lemma we know that such a proposal is accepted, then keeping the offers to $k$ out of those who vote for the proposal and offering the rest 0 is a profitable deviation.

\[\square\]

Lemma 3. In any Markovian equilibrium, for any $t$, any $s^t_4 \in S^t_4$, consider the subgame starting in stage 4 (proposal-making stage) of round $t$ with state $s^t_4$, if $x \in X$ and $x' \in X$, $x \neq \ldots$\(^{14}\)This continuation value is completely determined before the round-$t$ proposal-making and voting, because the recognition rule does not depend on the last period’s proposal or the vote on that proposal.

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are two proposals made in equilibrium with positive probability, define \( y \equiv \max_{i \neq p}(s_i) x_i \) and \( y' \equiv \max_{i \neq p}(s_i') x_i' \), i.e. \( y \) and \( y' \) are the largest share the proposer offers to a legislator other than herself in \( x \) and \( x' \), respectively, then \( y = y' \).

**Proof.** Suppose not and suppose without loss of generality that \( y > y' \). Then the legislator who gets \( y \) in the proposal \( x \), denoted by \( i^* \), must not vote yes for proposal \( x' \), because a yes vote from this legislator \( i^* \) would require at least a share of \( y \) (otherwise offering \( y \) to \( i^* \) in \( x \) would not have been optimal for the proposer). Define \( L \) and \( L' \) as the set of legislators (other than the proposer) who vote yes for the proposal \( x \) and \( x' \), respectively, then \( i^* \not\in L' \).

If \( |L| > k \), then obviously \( x \) is not optimal for the proposer - a profitable deviation would be offering 0 to \( i^* \) while keeping everyone else’s the same, this will lose one vote (that of \( i^* \)) but since \( |L| > k \), \( |L| - 1 > k \), so this new offer would still be accepted. Therefore it must be the case that \( |L| = k \).

Since \( i^* \in L \), \( i^* \not\in L' \), \( |L| = k \) and \( |L'| \geq k \), there exists \( j \in L' \) such that \( j \notin L \). Now since \( y > y' \) and \( y' \geq x'_j \) for all \( i \in N \), \( y > x'_j \). Consider a proposal \( x'' \) such that \( x''_i = x_i \) for all \( i \notin \{i^*, j\} \), \( x''_i = 0 \) and \( x''_j = x'_j \). This offered will be accepted (by \( (k-1) \) legislators from \( L \) plus \( j \)) and the proposer’s share for herself strictly increases (because \( x_i^* = y > x'_j = x''_j \)).

Therefore \( x \) cannot be optimal for the proposer \( p^t(s_i) \) and thus cannot be offered in equilibrium with positive probability, we have a contradiction. Therefore \( y = y' \).

\[ \square \]

**Proof of Proposition 1.** For arbitrary \( \epsilon > 0 \) and arbitrary \( s_0^t \in S_0^t \), if \( y^0(s_0^t) \leq \epsilon \), then \( y^0(s_0^t) \leq \max\{\epsilon, \frac{\delta}{1-\delta} \epsilon \} \).

We now consider the case when \( y^0(s_0^t) > \epsilon \), and we want to show that in this case it must be that \( y^0(s_0^t) \leq \frac{\delta}{1-\delta} \epsilon \).

For any \( t \), define \( L^t(s_0^t) \) as the set of legislators who vote for \( x^t \) at stage \( s_0^t \) (in stage 5 of period \( t \)) on the equilibrium path. Since \( y^0(s_0^t) > \epsilon > 0 \), following the same argument as in the proof of Lemma 3 we know that \( |L^t(s_0^t)| = k \). Also define \( \tilde{L}^t(s_0^t) = L^t(s_0^t) \cup \{p^t\} \), and
\[ \hat{L}^t(s_5^t) \equiv \hat{L}^t(s_5^t) \setminus \{i|x_i^t(s_5^t) = y_i^t(s_5^t)\}, \] where \( y_i^t(s_5^t) = \max_{i \neq p} x_i^t(s_5^t). \)  \(^{15}\) In words, \( \hat{L}^t(s_5^t) \) is the set of legislators who vote yes for the proposal at state \( s_5^t \) (including the proposer herself), and \( \hat{L}^t(s_5^t) \) is the same set only excluding the (non-proposer) legislator(s) who get(s) the highest share.

Since \( y^0(s_4^0) > \epsilon \), we have \( \forall j \notin \hat{L}^0(s_4^0), \delta V_j^1(s_4^0) \geq y^0(s_4^0) > \epsilon. \)  \(^{16}\) Note that \( |\hat{L}^t(s_5^t)| = \hat{L}^t(s_5^t) + 1 - |\{i|x_i^t(s_4^t) = y_i^t(s_4^t)\}| \leq k \), therefore the total number of \( j \)'s not in \( \hat{L}^0(s_4^0) \) is greater than or equal to \( n - k \).

Now because the recognition process exhibits one-period \( \epsilon \)-predictability of degree \( m < n - k \), this implies that among such \( j \)'s, there exists \( j^* \) such that \( j^* \) is not among the \( m \) legislators (therefore has less than \( \epsilon \) probability of being recognized next period, no matter what the next period political maneuverings are). The continuation value of \( j^* \) in the next period is

\[
V_j^1(s_4^0) = \Pr(\text{Being recognized}) \cdot \E(\text{Share when recognized}) \\
+ \Pr(\text{Not being recognized}) \cdot \E(\text{Share when not recognized}) \\
\leq \epsilon + \Pr(\text{Not being recognized}) \cdot \E(\text{Share when not recognized})
\]

This implies that \( \E(\text{Share when not recognized}) \geq V_j^1(s_4^0) - \epsilon \geq \frac{y^0(s_4^0)}{\delta} - \epsilon. \) And this means that there exists a path of states after which the proposer at period 1 gives \( j^* \) a share greater than or equal to \( \frac{y^0(s_4^0)}{\delta} - \epsilon. \) Recall we defined \( y_i^t(s_4^t) \) as the largest share a period-\( t \) proposer will give to a legislator other than herself, then this implies that there exists \( s_4^1 \in S_4^1 \) such that \( s_4^1 = (s_4^0, p^1, \theta^1, g^1, I^1) \) for some \( p^1, \theta^1, g^1 \) and \( I^1 \), such that \( y^1(s_4^1) \geq \frac{y^0(s_4^0)}{\delta} - \epsilon. \)

Following the same logic, we can show that there exist a series of states \( \{s_4^t\}_t \), where for every \( t, s_4^{t+1} = (s_4^t, p^{t+1}, \theta^{t+1}, g^{t+1}, I^{t+1}) \) for some \( p^1, \theta^1, g^1 \) and \( I^1 \), and \( y^{t+1}(s_4^{t+1}) \geq \frac{y^t(s_4^t)}{\delta} - \epsilon \) for all \( t \geq 0 \). Now note that \( \frac{y^t(s_4^t)}{\delta} - \epsilon \geq y_i^t \) if and only if \( y_i^t(s_4^t) \geq \frac{\epsilon \delta}{1-\delta}. \)

\(^{15}\) Again, for the case in which the equilibrium proposal-making involves mixed strategy, Lemma 3 shows that for any two proposals a proposer offers with positive probability in a Markovian equilibrium, the highest shares these two proposals give to a legislator other than the proposer herself must be equal. Therefore \( y_i^t(s_4^t) \) as defined above can also be used in a mixed-strategy equilibrium.

\(^{16}\) We do not need to explicit consider the (dis)utility from next-period political maneuvering as the payoff from political maneuvering is always non-positive, thus does not affect the sign of the above inequality.
Now suppose to the contrary that $y^0(s^0_4) > \frac{\epsilon \delta}{1 - \delta}$. Then from above we have $y^1(s^1_4) \geq \frac{y^0(s^0_4)}{\delta} - \epsilon > y^0(s^0_4)$, i.e. $y^1(s^1_4) > y^0(s^0_4)$. Therefore $y^1(s^1_4) > \frac{\epsilon \delta}{1 - \delta}$, and then we have $y^2(s^2_4) > \frac{y^1(s^1_4)}{\delta} - \epsilon \geq y^1(s^1_4)$. Iterating this, we can show that $\{y^t(s^t_4)\}$ is an increasing sequence. Since $\{y^t(s^t_4)\}$ is also bounded from above (by 1), it must have a limit.

Pick $\lambda$ such that $\frac{\lambda \delta}{1 - \delta} = y^1(s^1_4) - \frac{\epsilon \delta}{1 - \delta}$, then $\lambda > 0$ as $y^1(s^1_4) > y^0(s^0_4) > \frac{\epsilon \delta}{1 - \delta}$. Therefore, there exists $T^*$ such that for all $t > T^*$, $0 < y^{t+1}(s^{t+1}_4) - y^t(s^t_4) < \lambda$. That is, $y^{t+1}(s^{t+1}_4) < \lambda + y^t(s^t_4)$.

Therefore, we have $\lambda + y^t(s^t_4) > y^{t+1}(s^{t+1}_4) \geq \frac{y^t(s^t_4)}{\delta} - \epsilon$, which can be rearranged into $y^t(s^t_4) < \frac{\lambda \delta}{1 - \delta} + \frac{\epsilon \delta}{1 - \delta} = y^1(s^1_4)$ for all $t > T^*$. A contradiction to $\{y^t(s^t_4)\}$ being an increasing sequence.

Therefore, if $y^0(s^0_4) > \epsilon$ we must have that $y^0(s^0_4) < \frac{\delta}{1 - \delta} \epsilon$.

Therefore, for any $s^0_4 \in S^0_4$, we have $y^0(s^0_4) \leq \max\{\epsilon, \frac{\delta}{1 - \delta} \epsilon\}$.

\[\square\]

**Proof of Corollary 1.** It is straightforward to show the above strategy profiles constitute a Markovian equilibrium. Uniqueness follows directly from the above proof when $\epsilon = 0$.

\[\square\]

**Proof of Proposition 2.** For arbitrary $\epsilon > 0$ and arbitrary $s^0_4 \in S^0_4$, if $y^0(s^0_4) \leq \epsilon$, then $y^0(s^0_4) \leq \max\{\epsilon, \frac{\delta}{1 - \delta} \epsilon\}$.

We now consider the case when $y^0(s^0_4) > \epsilon$, and we want to show that in this case it must be that $y^0(s^0_4) \leq \frac{\delta}{1 - \delta} \epsilon$.

As in the proof of Proposition 1, for any $t$, define $L^t(s^t_5)$ as the set of legislators who vote for $x^t$ at state $s^t_5$ (in stage 5 of period $t$) on the equilibrium path. Since $y^0(s^0_4) > \epsilon > 0$, following the same argument as in the proof of Lemma 3 we know that $|L^t(s^t_5)| = k$. Also define $\bar{L}^t(s^t_5) \equiv L^t(s^t_5) \cup \{p^t\}$, and $\hat{L}^t(s^t_5) \equiv \bar{L}^t(s^t_5) \setminus \{i \mid x^t_i(s^t_4) = y^t(s^t_4)\}$, where $y^t(s^t_4) = \max_{i \neq p^t} x^t_i(s^t_4)$.

\[17\] In words, $\hat{L}^t(s^t_5)$ is the set of legislators who vote yes for the proposal

\[17\] Again, for the case in which the equilibrium proposal-making involves mixed strategy, Lemma 3 shows that for any two proposals a proposer offers with positive probability in a Markovian equilibrium, the highest shares these two proposals give to a legislator other than the proposer herself must be equal. Therefore $y^t(s^t_4)$ as defined above can also be used in a mixed-strategy equilibrium.
at state $s^t_5$ (including the proposer herself), and $\hat{L}^t(s^t_5)$ is the same set only excluding the (non-proposer) legislator(s) who get(s) the highest share.

Now consider the stage 1 of the next period (period 1), the state for this stage is $s^1_1 = (h^1_P, h^1_G, I^1)$, and this is precisely $s^0_4$. In general, for any path of states, we have $s^{t+1}_t = s^t_4$, and this implies that, in any pure strategy Markovian equilibrium, at the stage 4 and stage 5 of any period $t$ (i.e. the proposal-making and voting stages), the exact next-period political maneuvering activities of all legislators, $g^{t+1}$, are known to all legislators. Therefore, as the recognition process exhibits one-period conditional $\epsilon$-predictability of degree $m < n - k$, at stage 4 of any period $t$, the legislators all can identify a group of $m$ legislators with next-period recognition probabilities greater than $\epsilon$ while all others have only small ($\leq \epsilon$) probabilities of being recognized in period $t + 1$.

After establishing this, the rest of the proof is just like the proof of Proposition 1. Since $y^0(s^0_4) > \epsilon$, we have $\forall j \notin \hat{L}^0(s^0_4)$, $\delta V^1_j(s^0_4) \geq y^0(s^0_4) > \epsilon$. 18 Note that $|\hat{L}^t(s^t_5)| = |L^t(s^t_5)| + 1 - |\{ i | x^t_i(s^t_4) = y^t(s^t_4) \}| \leq k$, therefore the total number of $j$’s not in $\hat{L}^0(s^0_4)$ is greater than or equal to $n - k$.

As the recognition process exhibits one-period conditional $\epsilon$-predictability of degree $m < n - k$, at stage 4 of period 0, the legislators all can identify the group of $m$ possible next-period proposers, then among the above $j$’s, there exists $j^*$ such that $j^*$ is not among the $m$ legislators (therefore has less than $\epsilon$ probability of being recognized next period). The continuation value of $j^*$ in the next period is

$$V^1_{j^*}(s^0_4) = \Pr(\text{Being recognized})E(\text{Share when recognized}) + \Pr(\text{Not being recognized})E(\text{Share when not recognized}) \leq \epsilon + \Pr(\text{Not being recognized})E(\text{Share when not recognized}) \leq \epsilon + E(\text{Share when not recognized})$$

This implies that $E(\text{Share when not recognized}) \geq V^1_{j^*}(s^0_4) - \epsilon \geq \frac{y^0(s^0_4)}{4} - \epsilon$. And this means that there exists a path of states after which the proposer at period 1 gives $j^*$ a share.

18 We do not need to explicit consider the (dis)utility from next-period political maneuvering as the payoff from political maneuvering is always non-positive, thus does not affect the sign of the above inequality.
greater than or equal to $\frac{y^0(s^0_4)}{\delta} - \epsilon$. Recall we defined $y^t(s^t_4)$ as the largest share a period-$t$ proposer will give to a legislator other than herself, then this implies that there exists $s^1_4 \in S^1_4$ such that $s^1_4 = (s^0_4, p^1, \theta^1, g^1, I^1)$ for some $p^1, \theta^1, g^1$ and $I^1$, such that $y^1(s^1_4) \geq \frac{y^0(s^0_4)}{\delta} - \epsilon$.

Following the same logic, we can show that there exist a series of states $\{s^t_4\}_{t=0}^{\infty}$, where for every $t$, $s^{t+1}_4 = (s^t_4, p^{t+1}, \theta^{t+1}, g^{t+1}, I^{t+1})$ for some $p^t, \theta^t, g^t$ and $I^t$, and $y^{t+1}(s^{t+1}_4) \geq \frac{y^0(s^0_4)}{\delta} - \epsilon$ for all $t \geq 0$. Now note that $\frac{y^0(s^0_4)}{\delta} - \epsilon \geq y^t$ if and only if $y^t(s^t_4) \geq \frac{\epsilon^t}{1-\delta}$.

Now suppose to the contrary that $y^0(s^0_4) > \frac{\epsilon^0}{1-\delta}$. Then from above we have $y^1(s^1_4) \geq \frac{y^0(s^0_4)}{\delta} - \epsilon > y^0(s^0_4)$, i.e. $y^1(s^1_4) > y^0(s^0_4)$. Therefore $y^1(s^1_4) > \frac{\epsilon^1}{1-\delta}$, and then we have $y^2(s^2_4) > \frac{y^1(s^1_4)}{\delta} - \epsilon \geq y^2(s^2_4)$. Iterating this, we can show that $\{y^t(s^t_4)\}$ is an increasing sequence. Since $\{y^t(s^t_4)\}$ is also bounded from above (by 1), it must have a limit.

Pick $\lambda$ such that $\frac{\lambda}{1-\delta} = y^1(s^1_4) - \frac{\epsilon^0}{1-\delta}$, then $\lambda > 0$ as $y^1(s^1_4) > y^0(s^0_4) > \frac{\epsilon^0}{1-\delta}$. Therefore, there exists $T^*$ such that for all $t > T^*$, $0 < y^{t+1}(s^{t+1}_4) - y^t(s^t_4) < \lambda$. That is, $y^{t+1}(s^{t+1}_4) < \lambda + y^t(s^t_4)$.

Therefore, we have $\lambda + y^t(s^t_4) > y^{t+1}(s^{t+1}_4) \geq \frac{y^t(s^t_4)}{\delta} - \epsilon$, which can be rearranged into $y^t(s^t_4) < \frac{\lambda}{1-\delta} + \frac{\epsilon^t}{1-\delta} = y^t(s^t_4)$ for all $t > T^*$. A contradiction to $\{y^t(s^t_4)\}$ being an increasing sequence.

Therefore, if $y^0(s^0_4) > \epsilon$ we must have that $y^0(s^0_4) < \frac{\delta}{1-\delta} \epsilon$.

Therefore, for any $s^0_4 \in S^0_4$, we have $y^0(s^0_4) \leq \max \{\epsilon, \frac{\delta}{1-\delta} \epsilon\}$.}

\[\square\]

**Proof of Corollary 2.** It is straightforward to show the above strategy profiles constitute a Markovian equilibrium. Uniqueness follows directly from the above proof when $\epsilon = 0$.

\[\square\]

**Proof of Corollary 3.** Since the chair has a strict preference ordering over proposers, he is going to pick his favorite legislator as the proposer. Thus, the recognition process is one-period conditionally predictable of degree 1. Applying Corollary 1, the unique Markovian equilibrium outcome is $\epsilon_{\rho^0}$.

\[\square\]
Proof of Proposition 3. Define $y^t \equiv \max_{j \neq p^t} x^t_j$ as the largest share that period-$t$ proposer $p^t$ offers in equilibrium to a party other than her own. Similar to the proof of our main result, define $L^t$ as the set of parties who vote for the proposal in period $t$, $\tilde{L}^t \equiv L^t \cup \{p^t\}$, and $\hat{L}^t \equiv \tilde{L}^t \setminus \{j : x^t_j = y^t\}$. We denote $J^t \equiv \{1, 2, \ldots, P\} \setminus \hat{L}^t$, so $J^t$ is the set of parties who receives the largest share from the period-$t$ proposer plus the parties who are excluded by the period-$t$ proposer. Finally let $V^t_j$ denote the continuation value of a legislator in party $j$ in period $t$.

Suppose to the contrary that in equilibrium the largest party gives a strictly positive share to some party other than her own. That is $y^0 > 0$. For this proposal to be optimal for the first proposer (the largest party), we must have that for all $j \in J^0$, $\delta V^1_j \geq y^0 > 0$.

Since $n_1 < \frac{n}{2} - 1$, so that no party has a majority, following the same argument as in the proof of Lemma 2, we know that at any period, the set of parties that are excluded by the proposer is non-empty. Therefore, $|J^0| \geq 2$. This is because by definition, $J^0$ consists of the party (parties) other than the proposer who receives $y^0$, plus the party (parties) excluded by the proposer. As these two sets are disjoint and both non-empty, $J^0$ must contain at least two parties.

Among these two parties, at least one of them cannot be recognized in period 1. Pick one such party and denote it by $j^1$, then $x^1_{j^1} = V^1_{j^1}$. As we show above $\delta V^1_{j^1} \geq y^0 > 0$, we have $\delta y^1 \geq \delta x^1_{j^1} = \delta V^1_{j^1} \geq y^0$.

Following the same argument, we can show that $\{\delta^i y^t\}_{i=0}^\infty$ is an increasing sequence, which is bounded strictly above 0 and below 1. But for $\delta < 1$ this means that $y^t \not\to \infty$ as $\delta^t \searrow 0$. As $y^t \leq 1 \forall t$, a contradiction.

$\square$

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19For simplicity here we omit the arguments in all the notations we define.