Money versus Memory

Luis Araujo                  Braz Camargo
Michigan State University   University of Western Ontario

March 31, 2009

Abstract

Kocherlakota (1998) shows that money can be understood as a primitive form of memory in the following sense: the set of allocations that money can implement is contained in the set of allocations that memory can implement. In this paper we investigate a notion of memory that is more primitive than Kocherlakota’s and show that it unties technological properties of money which are combined in his notion. We obtain that the technological role of money lies not only on being a record of past actions, but also on being a record of past trading opportunities, a dimension of money that has been overlooked in the monetary literature. In particular, money is better than memory if memory is just a record of past actions.

1 Introduction

An important insight of monetary theory is that fiat money helps to keep track of past actions (Ostroy (1973), Lucas (1980), Townsend (1980), and Aiyagari and Wallace (1991)). Kocherlakota (1998) expands this point by showing that memory, appropriately defined, subsumes money in the following sense: any allocation that can be achieved with money can also be achieved with memory.\footnote{Kocherlakota (1998) establishes this result in a broad class of environments which includes all the environments commonly used in monetary theory.} Based on this result he concludes that, from a technological

1
standpoint, money is a primitive form of memory.

The notion of memory put forth by Kocherlakota is designed to reproduce the role of money as a record of past actions: he defines memory as knowledge on the part of an agent of the past history of his partners and all the agents that were directly or indirectly in contact with his partners. As such, this notion of memory contains an element of circularity that can potentially hinder a deeper understanding of the technological properties of money. Indeed, if one takes as a starting point the assumption that agents have the knowledge of past experiences that is required to replicate monetary balances, it is natural that this knowledge can be used to implement monetary allocations in the absence of money. In this article we investigate a more primitive notion of memory and show that it unties technological properties of money which are combined in Kocherlakota’s notion.

In any trading environment, an agent’s experience includes not only his past actions, but also the nature of his trading opportunities. For example, in random matching environments such as the ones of Shi (1995) and Trejos and Wright (1995), the information in an agent’s history includes which of his past meetings were single–coincidence meetings and which were not. We define memory as knowledge on the part of an agent of the last period action and type of his current partner, where an agent’s type in a period is a complete description of this agent’s trading opportunities in this given period. This definition of memory is weaker than Kocherlakota’s in the sense that it contains a strict subset of the information that is available under his notion.

We analyze three different settings, a random matching economy, an overlapping generations economy, and a turnpike economy, and show that the first–best can be achieved in these three environments if agents have access to memory. Intuitively, memory sustains non–autarkic allocations because it rewards the agents’ willingness to produce whenever it is socially beneficial to do so. This is in line with Kocherlakota’s assessment that money is a primitive form of memory. A more interesting implication of our analysis is that the knowledge of the previous period type of an agent plays an important role in our results. We show that money can be better than memory if an agent can only observe the past actions
of his partners (the observed actions need not be restricted to the previous period). The reason for this is that now memory cannot distinguish a situation in which an agent does not produce because his trading partners do not like his good from a situation in which an agent chooses not to produce to trading partners who like his good. This shows that the technological role of money lies not only on being a record of past actions, but also on being a record of past trading opportunities, a dimension of money that has been overlooked in the monetary literature.

The paper is organized as follows. In Section 2 we show that memory can achieve the first-best in a random matching economy. The analysis in the overlapping generations economy and in the turnpike economy is very similar, and for this reason it is relegated to the Appendix. In Section 3 we show that money can be superior to memory if the latter does not include the observation of past types. We conclude in Section 4.

2 A Random Matching Economy with Memory

The model is based on Shi (1995) and Trejos and Wright (1995). Time is discrete and the horizon is infinite. There is a continuum of mass one of infinitely-lived agents and a continuum of divisible goods. An agent cannot consume the good that he produces but he obtains positive utility from the consumption of a subset of goods produced by other agents. Precisely, an agent who in a period consumes \( y \) units of a desirable good and produces \( x \) units obtains utility \( u(y) - x \), where \( u : \mathbb{R}_+ \to \mathbb{R} \) is strictly increasing and strictly concave, \( u(0) = 0 \), the right-derivative of \( u \) at zero is greater than 1, and \( \lim_{x \to \infty} u(x) - x < 0 \).

The efficient amount of production is the unique \( x^* \) that maximizes \( u(x) - x \). Notice that \( x^* > 0 \) by assumption. Agents maximize their expected discounted utility with discount factor \( \beta \in (0, 1) \). In every period agents are randomly and anonymously matched in pairs. In a meeting between two agents, say \( i \) and \( j \), there is a probability \( \sigma \left( 0 < \sigma < \frac{1}{2} \right) \) that \( i \) likes the good produced by \( j \) but \( j \) does not like the good produced by \( i \), and a probability \( \sigma \) that \( j \) likes the good produced by \( i \) but \( i \) does not like the good produced by \( j \).
no double—coincidence meetings.

All agents have access to a technology that we label memory. This technology allows an agent to observe the “recent history” of his current partner, that is, the type and action of his current partner in the previous period. The action of an agent is the quantity of goods \( x \in \mathbb{R}_+ \) that he produces, while his type indicates whether he is a producer (\( p \)), i.e., he is able to produce the good desired by his partner; he is a consumer (\( c \)), i.e., he likes the good produced by his partner; or he is in a no-coincidence meeting (\( n \)). The history of an agent includes his past types and actions and the types and actions of all agents in meetings in which he participated. It also includes the recent history of his current partner as well as his past partners’ recent histories.

Since the economy is populated by a continuum of agents, there is no loss in generality if we assume that an agent does not use information about the history of his types and actions up to but not including the previous period. An agent also does not use information on the actions of his past partners as well as information about the recent history of his past partners. The reason is that this information is private to the agent and is independent of his current and future partner’s strategies. This implies that the strategy of an agent (say agent \( s \)) is given by \( \sigma^s = \{ \sigma^s_1 \} \), \( \sigma^s_1 : \Gamma \times (\Gamma \times \mathbb{R}_+)^2 \to \Delta (\mathbb{R}_+) \), where \( \Gamma = \{ \tau \mid \tau \in \{ p, c, n \} \} \) is the set of types and \( \Delta (\mathbb{R}_+) \) is the set of all probability distributions on \( \mathbb{R}_+ \).

In what follows, we assume that strategies are ex—ante symmetric and stationary. Hence, we can drop the subscript \( t \) and the superscript \( s \) in the description of a strategy. Finally, we restrict attention to strategies in which consumers and agents in a no—coincidence meeting never produce. This is consistent with our objective of constructing a strategy that implements the best possible allocation as an equilibrium outcome. In fact, the first best is achieved if an agent produces \( x^* \) if and only if he is a producer, otherwise he produces zero.\(^2\)

All in all, a strategy is henceforth summarized by a function \( \sigma : (\Gamma \times \mathbb{R}_+)^2 \to \Delta (\mathbb{R}_+) \), which describes the action of a producer after any pair of recent histories \( (\tau^s, x^s), (\tau, x) \in \Gamma \times \mathbb{R}_+ \).

Assume that agent \( s \) follows a strategy \( \sigma^s \) while all other agents follow a strategy \( \sigma \). In

\(^2\)We also assume that, in the first period, the history of an agent corresponds to the empty set (\( \emptyset \)).
every period, the utility of agent $s$ depends on the joint distribution of previous period types and actions of all agents in the economy. Let this distribution be given by $\mu = \{\mu_t\}$. Clearly, $\mu$ is induced by $\sigma$. However, $\mu$ does not depend on $\sigma^s$. This is so because each agent is negligible in a continuum of agents. Consistent with this observation, we also assume that the belief of agent $s$ with respect to $\mu$ is independent of his own history. In other words, because his personal history contains a finite number of observations, it cannot change the agent’s belief about the distribution of previous period types and actions. This assumption is in the spirit of sequential equilibrium for finite games.3

Let $U (\sigma^s, \sigma | (\tau^s, x^s), (\tau, x), \mu)$ be the utility of agent $s$ if he follows a strategy $\sigma^s$ and all other agents follow a strategy $\sigma$, conditional on the previous period type and action of agent $s$ being $(\tau^s, x^s)$, and on the recent history of his current partner being $(\tau, x)$. A strategy $\sigma^*$ is an equilibrium if, for all $(\tau^s, x^s), (\tau, x) \in \mathbb{R}_+ \times \Gamma$, and every strategy $\sigma$,

$$U (\sigma^*, \sigma^* | (\tau^s, x^s), (\tau, x), \mu^* ) \geq U (\sigma, \sigma^* | (\tau^s, x^s), (\tau, x), \mu^* )$$

where $\mu^*$ is induced by $\sigma^*$.

We sustain the first best as an equilibrium outcome with a strategy in which the action of an agent only depends on his current type and on the recent history of his current partner, that is, it does not depend on his personal history (his own past type and action).4 As a result, the utility of an agent is independent of the recent history of his current partner and is independent of the joint distribution of previous period types and actions of all agents in the economy. This greatly simplifies the analysis. Consider then the following candidate strategy. If an agent is a producer, he produces $x^*$ with probability $q$ and 0 with probability $1 - q$ if and only if his current partner was a producer in the previous period and produced a quantity $x \neq x^*$. Otherwise, he produces $x^*$ with probability 1.5 Clearly, this candidate strategy implements the first best.

---

3 See Takahashi (2008) for a discussion of this point.

4 Takahashi (2008) also considers equilibria where strategies are independent of an agent’s personal history. See his paper for a discussion of how this type of equilibria relate to belief-free equilibria.

5 As stated above, if an agent is not a producer, he always chooses $x = 0$. 

5
Consider the expected payoff of a producer under the candidate strategy. If he meets an agent with a history that is on the equilibrium path, his expected payoff is 
\[-(1 - \beta) x^* + \beta V_g,
\]
where \(V_g\) is the expected payoff of an agent with a personal history that is on the equilibrium path. If, instead, he meets an agent with a history that is off the equilibrium path, his expected payoff is 
\[q[-(1 - \beta) x^* + \beta V_g] + (1 - q) \beta V_b,\]
where \(V_b\) is the expected payoff of an agent with a personal history that is off the equilibrium path. Assume that
\[-(1 - \beta) x^* + \beta V_g = \beta V_b.\]  
If this condition holds, a producer is indifferent between producing \(x^*\) and producing 0. It also implies that his expected payoff is equal to the expected payoff associated with the best possible deviation from the candidate strategy, i.e., \(\beta V_b\) (the agent does not produce and obtains \(V_b\) as continuation payoff).

When (2) holds, it is straightforward to compute the expected payoff under the candidate strategy after any possible history. We obtain that
\[V_g = \sigma \{\beta V_b + [(1 - \beta) u(x^*) + \beta V_g]\} + (1 - 2\sigma) \beta V_g,\]  
and that
\[V_b = \sigma \{\beta V_b + [(1 - \beta) qu(x^*) + \beta V_g]\} + (1 - 2\sigma) \beta V_g.\]  
For instance, (3) can be interpreted as follows. There is a probability \(\sigma\) that the agent will be a producer. In this case, irrespective of the personal history of his partner, his continuation payoff is \(\beta V_b\). There is a probability \(\sigma\) that he will be a consumer. In this meeting, he consumes \(x^*\) with probability 1. In turn, because his personal history is consistent with the equilibrium path, his continuation payoff is \(V_g\). Finally, there is a probability \((1 - 2\sigma)\) that he participates in a no-coincidence meeting. In this meeting, he does not consume or produce. Again, because his personal history is consistent with the equilibrium path, his continuation payoff is \(V_g\). A similar interpretation applies to (4).

Consider (2) one more time. If we substitute (3) and (4) into this expression, we obtain that there exists a unique value of \(q\) under which it is satisfied, namely
\[q = 1 - \frac{x^*}{\beta \sigma u(x^*)}.\]
Note that $q$ is always below 1, and it is greater than zero as long as

$$\beta > \frac{x^*}{\sigma u(x^*)}. \quad (6)$$

In summary, if $q$ satisfies (5) and agents are sufficiently patient (that is, condition (6) is satisfied), irrespective of his history, a producer is indifferent between producing the efficient quantity $x = x^*$ and producing $x = 0$. Hence, he is always willing to go along with the candidate strategy.

We have thus constructed an equilibrium with two main features. First, an agent’s current action does not depend on his past actions. Second, irrespective of his history, an agent is indifferent between producing the efficient quantity $x = x^*$ and producing $x = 0$. These features allow us to use memory as a mechanism that rewards an agent with a “good” personal history (by producing the efficient quantity with probability 1) and punishes an agent with a “bad” personal history (by producing zero with a positive probability). Note that the punishment for a bad personal history cannot be too large ($q$ cannot be too small), otherwise an agent would have an strict incentive to produce the efficient quantity $x^*$ even if his partner carries a bad personal history. Proposition 1 summarizes our result.

**Proposition 1.** If $\beta > \frac{x^*}{\sigma u(x^*)}$, memory sustains the first–best as an equilibrium outcome.

It is straightforward to extend the above proposition to any (symmetric) allocation where in every single–coincidence meeting the producer transfers $x \in (0, x^*)$ units of the good to the consumer. It is interesting to compare the result in Proposition 1 with Proposition 2 in Kocherlakota (1998). Proposition 2 in Kocherlakota (1998) demonstrates that any monetary allocation can be replicated by a technology that allows an agent to observe not only the trading history of his partners, but also the trading history of all agents that were directly or indirectly in contact with his partners. Our notion of memory cannot be used to replicate monetary allocations. However, if $v$ is a payoff that the agents obtain in some (symmetric) monetary equilibrium, then memory can sustain an allocation where the agents obtain exactly the same payoff.
3 Money Better than Memory

The definition of memory put forth in this paper includes not only the observability of an agent’s action in the previous period but also the observability of his type in the previous period. The observability of past actions is clearly an essential component: if an agent cannot observe the action of his current partner in the previous period, autarky is the only possible equilibrium in the absence of money. The reason is that an agent’s current decision has no effect on his continuation payoff.

What happens if we assume that past actions are observable but past types are not observable? In a random matching model, this implies that, upon observing the action of his current partner in the previous period, an agent does not know whether the latter was a producer, a consumer, or neither. As a result, the first best cannot be achieved as an equilibrium outcome. The argument runs as follows. Assume that the first-best is implemented, that is, an agent produces the efficient quantity $x^*$ if and only if he is paired with another agent that likes his good. If types are private information, this behavior can only be sustained in equilibrium if the expected payoff of producing $x^*$ is exactly equal to the expected payoff of producing zero. Indeed, an agent can always imitate the behavior that produces the highest expected payoff. This implies that, if an agent produces zero, his expected continuation payoff must be strictly lower than the expected continuation payoff of an agent producing $x^*$. The problem is that, in the first best, the expected continuation payoff on the equilibrium path cannot depend on the agent’s history, which leads to a contradiction. It is important to point out that this argument applies not only to our definition of memory. In fact, if past types are not observable, the first best cannot be achieved even if we assume that each agent knows the past actions of his partners, and the past actions of all agents that were directly or indirectly in contact with his partners.

The first best can be implemented when an agent observes the type of his partner in the previous period because the expected payoff of an agent that does not produce when he meets another agent that does not like his good is strictly higher than the expected payoff.
of an agent that produces the efficient quantity $x^*$. It is true that an agent who does not produce to another agent that likes his good is punished with an expected continuation payoff inferior to the one achieved on the first best. However, this behavior does not arise on the equilibrium path.\footnote{The assumption that types are observable does not enter in the definition of memory in the overlapping generations model and in the turnpike model (see the Appendix). Naturally, one can extend these models and assume that young agents (in the OLG model), stayers and mover (in the turnpike model) have different production types. Under this extension, if past types are not observable, it is also the case that the first best cannot be achieved in equilibrium. Hence, the importance of including the observability of past types in the definition of memory is not restricted to random matching models.}

What allocations can be implemented with memory when types are private information? In the previous section we considered a strategy where memory is used as a mechanism that rewards agents willingness to exert effort. If types are not observable, a natural strategy is one in which memory rewards agents observed effort. Precisely, the candidate strategy is as follows.\footnote{If types are not observable, a history is just the production $x$ of the agent’s current partner in the previous period.} An agent produces $x' > 0$ with probability $q'$ and 0 with probability $1 - q'$ if and only if his current partner produced a quantity $x \neq x'$ in the previous period. Otherwise, he produces $x'$ with probability 1. A direct adaptation of the proof in section 2 shows that this strategy is an equilibrium as long as $\beta > \frac{x'}{\sigma u(x')}$. Moreover, an agent’s expected payoff is $\sigma u(x') - x'$, which is strictly smaller than the payoff achieved under the first best, $\sigma [u(x^*) - x^*]$.

In summary, if past types are not observable, there still exists equilibria in which agents produce along the path of play. However, these equilibria are inefficient. This inefficiency can only be ameliorated if there is a technology that allows an agent to credibly convey his current type to his future partner. Our definition of memory satisfies this requirement. Money is another mechanism that satisfies this requirement, an aspect that has been mostly overlooked in the literature. In a monetary equilibrium, an agent’s monetary balance reveals information about his past types. For instance, if an agent holds money, it must be the case that, at some point in the past, he produced to another agent who liked his good. This information cannot be credibly conveyed if only past actions are observable. Hence, if
memory is to achieve the benefits of money, it must include information on the observability of past types. A direct implication is that money can be interpreted as a primitive technology that credibly discloses such information.

The reasoning above naturally leads to the following question: what allocations can be achieved with money if past types are not observable? In the context of random matching models, Kocherlakota (2002) shows that the first best can be achieved using two perfectly divisible monies. This implies that money is better than memory if past types are not observable.

4 Conclusion

If memory is designed to imitate money, a natural implication is that money is a primitive form of memory. In this article, we define memory simply as a technology that reveals the recent histories of agents in a match. We prove that, even though this technology cannot replicate monetary balances, it can achieve the first best. Intuitively, memory allows an agent to be rewarded by his willingness to exert effort in the recent past. An interesting implication of our result is that memory must include not only recent actions but also recent types if it is to achieve the benefits of money. This shows that money is not only a primitive record-keeping technology, but also a technology that discloses private information about past trading opportunities.

References


---

8 The mechanism that supports the first best constructs a one to one mapping between individual histories and the decimal expansions of money holdings. This mapping allows money to become a recording device that keeps a perfect track of an agent’s past actions.

9 Perfect divisibility is not required in a standard overlapping generations model or in a standard turnpike model. See the Appendix.


Appendix

A. Overlapping Generations with Memory

Time is discrete and the horizon is infinite. In every period, a generation is born, which comprises a unit continuum of individuals who live for two periods. In the first period of his life an individual is young and in the second period he is old. In period 1, there is also a generation of agents, called the initial old. Young agents can produce a divisible and perishable good while old agents are unproductive. Moreover, only old agents derive utility from consumption. Precisely, the utility of an agent who consumes \( x \) units of goods when old and produces \( y \) units of good when young is given by \( u(x) - y \). We assume that \( u : \mathbb{R}_+ \rightarrow \mathbb{R} \) satisfies the same properties as in the random matching model, and we let \( x^* \) denote the efficient amount of production. In every period each young agent is randomly matched with an old agent. All young agents have access to a technology that allows them to observe the previous period action of the old agent they are currently matched with.

Actions and histories are defined as in the random matching model. The first best is implemented when each young agent produces the quantity \( x^* \) to an old agent. First of all, there is no need to specify the action of an agent when old because old agents are unproductive. We sustain the first best as an equilibrium outcome with a strategy in which the action of an young agent only depends on the action of his current match when he was young. This implies that the set of relevant histories is \( H = \emptyset \cup \{x : x \geq 0\} \), where \( x \) is the quantity of goods produced by an old agent in the previous period. We added \( \emptyset \) to describe the history of the initial generation of old agents. Consider the following candidate strategy. A young agent chooses \( x = x^* \) with probability 1 if \( h \in \{\emptyset, x^*\} \). If, instead, \( h \notin \{\emptyset, x^*\} \), he chooses \( x = x^* \) with probability \( q \) and \( x = 0 \) with probability \( 1 - q \). On the path of play, this strategy implements the first best.

We focus on the expected payoff of a young agent under the candidate strategy. If he is matched with an old agent with a history \( h \in \{\emptyset, x^*\} \), his payoff is \( -x^* + u(x^*) \). If, instead, he is matched with an old agent with a history \( h \notin \{\emptyset, x^*\} \), his expected payoff is
\[ q \left[ -x^* + u(x^*) \right] + (1 - q) qu(x^*). \] Assume that

\[ -x^* + u(x^*) = q \left[ -x^* + u(x^*) \right] + (1 - q) qu(x^*). \tag{7} \]

We can rewrite (7) as

\[ q = 1 - \frac{x^*}{u(x^*)} \in (0, 1). \tag{8} \]

When \( q = 1 - \frac{x^*}{u(x^*)} \), a young agent is indifferent between producing the efficient quantity and not producing anything. As a result, he is willing to punish an old agent with a bad history by producing zero units with a positive probability, and he is willing to reward an old agent with a good history by producing the efficient quantity. This ensures that, on the equilibrium path, the efficient quantity is always produced.

**B. Turnpike with Memory**

This model is based on Townsend (1980). Time is discrete and the horizon is infinite. There is an infinite number of trading posts in the economy, located at the integer points on the real line. Agents can be of two types, “stayers” and “movers”. Stayers always stay at the same trading post while movers move 1 trading post to the right at the end of every period. In period 1, there are \( N \) stayers and \( N \) movers in each trading post. Stayers (movers) are endowed with one (zero) unit of a divisible and perishable good if \( t \) is odd and zero (one) unit if \( t \) is even. The instantaneous utility of an agent that consumes \( x \) units of goods is \( u(x) \), where \( u : \mathbb{R}_+ \rightarrow \mathbb{R} \) is strictly increasing and concave, \( u(0) = 0 \). In every period each stayer is randomly paired with a mover. Each stayer (mover) has access to a technology that allows him to observe the previous period action of the mover (stayer) he is currently paired with. Agents maximize their expected discounted utility with discount factor \( \beta \in (0, 1) \).

The action of an agent is the quantity of endowment that he gives to the agent he is currently paired with. Histories are then defined as in the random matching model. The first best is implemented when each agent consumes \( \frac{1}{2} \) units of goods in every period. This implies that in odd (even) periods, stayers (movers) give half of their endowment to movers (stayers). Note that there is no need to specify the actions of stayers in even periods and
the actions of movers in odd periods, because they have no endowment. We sustain the
first best as an equilibrium outcome with a strategy in which the action of an agent only
depends on the previous period action of his current match. This implies that the set of
relevant histories is \( H = \varnothing \cup \{ x : x \in [0,1] \} \), where \( x \) is the quantity of endowment offered
by a stayer (mover) to his previous match, if the match took place in an odd (even) period.
We let \( \varnothing \) denote the history of all agents at the beginning of period 1, and also the histories
of stayers (moves) in even (odd) periods, that is, periods in which they have no endowment.
Consider the following candidate strategy. In every odd (even) period, a stayer (mover)
choose \( x = \frac{1}{2} \) with probability 1 if \( h \in \{ \varnothing, \frac{1}{2} \} \). If, instead, \( h \notin \{ \varnothing, \frac{1}{2} \} \), he chooses \( x = \frac{1}{2} \) with probability \( q \) and \( x = 0 \) with probability \( 1 - q \). On the path of play, this strategy
implements the first best.

Let \( V_h \) be the agent’s continuation payoff after a history \( h \), and consider the expected
payoff of a stayer (mover) an odd (even) period. If he is matched with an agent with a history
\( h \in \{ \varnothing, \frac{1}{2} \} \), his payoff is \((1 - \beta) \left[ u \left( \frac{1}{2} \right) + \beta u \left( \frac{1}{2} \right) \right] + \beta^2 V_{\varnothing} \). If, instead, he is matched with an agent with a history \( h \notin \{ \varnothing, \frac{1}{2} \} \), his expected payoff is \( q \{ (1 - \beta) \left[ u \left( \frac{1}{2} \right) + \beta u \left( \frac{1}{2} \right) \right] + \beta^2 V_{\varnothing} \} +
(1-q) \{ (1-\beta) \left[ u \left( 1 \right) + \beta qu \left( \frac{1}{2} \right) \right] + \beta^2 V_{\varnothing} \} \). Assume that
\[
\frac{u \left( \frac{1}{2} \right) + \beta u \left( \frac{1}{2} \right)}{u \left( 1 \right) + \beta qu \left( \frac{1}{2} \right)} = \frac{u \left( 1 \right) + \beta qu \left( \frac{1}{2} \right)}{u \left( \frac{1}{2} \right)}.
\] (9)
We can rewrite (9) as
\[
q = 1 - \frac{u \left( 1 \right) - u \left( \frac{1}{2} \right)}{\beta \left( \frac{1}{2} \right)}.
\] (10)
Note that \( q \) is always below 1, and it is greater than zero as long as
\[
\beta > \frac{u \left( 1 \right) - u \left( \frac{1}{2} \right)}{u \left( \frac{1}{2} \right)}.
\] (11)
When \( q = 1 - \frac{u \left( 1 \right) - u \left( \frac{1}{2} \right)}{\beta u \left( \frac{1}{2} \right)} \), an agent is indifferent between producing sharing and keeping
his endowment. He is thus willing to punish an agent with a bad history by keeping his
endowment with a positive probability, and he is willing to compensate an agent with a
good history by sharing his endowment.