Fraternities and Labor Market Outcomes

Sergey V. Popov
Department of Economics
University of Illinois
popov2@uiuc.edu

Dan Bernhardt
Department of Economics
University of Illinois
danber@ad.uiuc.edu

Draft: September 16, 2008

Abstract

We model how the choices by students to “rush” a fraternity, and the choices by a fraternity of whom to admit, interact with the signals that firms receive about student productivities to determine labor market outcomes. Both the fraternity and students care about future wages and fraternity socializing values. We first show that if the signals firms receive about students are either perfectly informative or perfectly noisy, then fraternity membership has no impact on labor market outcomes. For intermediate signaling technologies, however, three types of equilibria can exist: pessimistic beliefs by firms about the abilities of fraternity members can support an equilibrium in which no one pledges; optimistic beliefs can lead to higher wages for fraternity members than non-members, so that in equilibrium everyone whom the fraternity would like to admit actually pledges; and an equilibrium in which most fraternity members have intermediate abilities—less able students apply, hoping to be mixed in with better students, but are rejected unless they have high fraternity socializing values, while most very able students do not apply to avoid being tainted in labor market outcomes due to being mixed in with less able fraternity members. We take the model to the data, and show that this latter equilibrium can reconcile the ability distribution of fraternity members at the University of Illinois.

*We thank George Deltas, Odilon Camara and participants at the Midwest Economics Association Annual Conference (2008) for helpful comments. We especially thank the Fraternity and Sorority Affairs staff and Division of Management Information staff of the Office of the Dean of Students of University of Illinois for assistance with data collection. All errors are ours.
1 Introduction

To many, the word “fraternities” brings to mind images of beer, parties and fun. Yet, fraternity membership also enters prominently the job seeking process of many students: resumes often devote scarce space to highlighting a student’s society memberships in addition to the standard information about education, work experience, awards, etc. This suggests that fraternity membership helps employers evaluate a person’s productivity. On first impression it is not clear why fraternity or sorority membership should matter for labor market outcomes. In particular, while fraternities make significant time demands — members must spend considerable time picking up trash on highways, raising money for charitable causes, and so on — these activities appear largely unrelated to skill development for future careers. Nonetheless, fraternities draw many applicants who are eager spend money and devote time on these activities, and employers seem to weigh membership information positively.

We develop a theory of fraternity membership and filtering by firms that makes sense of these observations. Students are distinguished by a fraternity socializing value and their productivity as a worker. Fraternities value both the future wages generated by members and their socializing values. Firms combine information in noisy signals about student productivities with fraternity membership status to set wages. To emphasize the key economic forces, we suppose that fraternity socializing values are not directly valued by firms and are uncorrelated with worker productivities. We further assume away all standard club features for fraternities as in Buchanan (3), so that there are no consumption spillovers due to the presence of other students. So, too, we assume away any networking externalities or services that a fraternity might provide. As a result, the fraternity membership statuses of other students only affect outcomes via the equilibrium beliefs that firms form about the distribution of abilities of fraternity members and non-members.

We first identify sufficient conditions for fraternity membership not to matter for job market outcomes. In particular, we show that if the signal that firms receive about a student’s productivity is either perfectly informative or perfectly noisy, then equilibrium wages do not depend on a student’s fraternity membership. As a result, whether a student rushes a fraternity depends only on his
fraternity socializing value. If signals are perfectly informative, fraternities trade off between productivity and socializing value in admission, but a student’s wage will equal his known productivity, rendering membership irrelevant for labor market outcomes. If, instead, signals are perfectly noisy, a fraternity would like to commit to excluding low ability students with high socializing value, and to accepting high ability students with low socializing value. However, with perfectly noisy productivity signals, firms have no source other than fraternity membership for evaluating a student’s ability, so that fraternities weigh only socializing values in admission. As a result, fraternity membership conveys no information to firms about ability, so that wages do not hinge on fraternity membership.

In sum, we show that for fraternity membership to affect job market outcomes, firms must receive signals about a student productivities that are noisy, but not perfectly so. Then, because more productive students tend to earn higher wages, fraternities trade off between productivity and fraternity socializing values when deciding which pledge applicants to accept. In particular, fraternities accept students with low socializing values who are sufficiently able. Students may face a different type of trade off—more able students may incur a labor market cost from joining a fraternity, as their fraternity membership may lump them in with intermediate quality students, making it harder for the able students to distinguish themselves in the eyes of firms. In such a situation, sufficiently more able students may be reluctant to pledge fraternities, so that among students with high ability, only those with higher fraternity socializing values pledge (even though the fraternity would happily accept some able students who do not apply).

We then turn to a three-signal setting in which we can explicitly solve for the multiple equilibria that emerge in the fraternity game. We identify three types of equilibria: (a) an “empty fraternity” equilibrium in which no student applies to the fraternity, supported by beliefs of firms that any student who joins the fraternity is especially lacking in ability; (b) an equilibrium in which most fraternity members are of intermediate ability—less able students apply, hoping to be mixed in with better students, but are rejected unless they have high fraternity socializing values, while very able students who do not have very high fraternity socializing values do not apply to avoid being mixed in with less able fraternity members; and (c) an equilibrium in which employer beliefs about the abilities of fraternity members are more optimistic—so that fraternity membership would increase the expected wage of each student type. In these latter two equilibria, relatively low ability students expect higher wages if they gain fraternity membership than if they do not; while in the
second equilibrium, but not the third, higher ability students expect lower wages if they join. That is, fraternity membership may taint labor market outcomes for high ability students, but not low.

In this three signal setting, we then investigate whether equilibria of our model are consistent with the data. Specifically, to do this, we obtain data on cumulative GPAs of seniors at the University of Illinois for fraternity members and non-members. Using GPA as a noisy indicator of ability, we find that an equilibrium in which most fraternity members are of intermediate ability—where high ability students are tainted by membership in fraternities—can generate a distribution over probability of membership conditional on ability that closely mirrors the distribution over the probability of fraternity membership conditional on GPA found in the data. We back out estimates of primitives—the time costs of fraternity participation, and the tradeoffs of both students and fraternity between socializing values and future wages.

We next review the literature and then provide a brief overview of fraternities. In section 2 we develop our model and analysis. Section 3 considers a three signal setting. Section 4 concludes. Some proofs are in an appendix.

1.1 Related Literature

Our model and analysis can be integrated into the endogenous statistical discrimination literature. The closest papers are Moro and Norman (8) and Austen-Smith and Fryer (2). Moro and Norman (8) consider a setting in which individuals choose whether to make a costly investment in education, when firms receive noisy signals of that investment.\(^1\) They show how a productivity irrelevant aspect such as racial identity can affect investment choices if firms believe that one population is more likely to invest. In particular, one can support multiple equilibria, one where beliefs do not depend on race, and one where they do.\(^2\) So, too, in our economy, the essence is how beliefs of firms about the abilities of workers who generate different signals are affected by the equilibrium fraternity membership status. One central difference between our economy and this literature is that race is exogeneous, while fraternity membership is endogenous, and, moreover, fraternity

\(^1\)See also Coate and Loury (4), Fang and Norman (5) and Norman (9) for related models.
\(^2\)We could augment our economy to have firm beliefs depend on race as well, with the result that race will enter both the decision of whether to apply to a fraternity, and the decision by the fraternity of whether to accept an applicant.
\(^3\)Mailath, Samuelson and Shaked (6) develop a related search model in which firms choose which populations to search, and each population makes investments in skills based on beliefs about firm search intensities. Again, asymmetric search intensities can give rise to asymmetric investment choices in two otherwise identical populations.
membership is not solely determined by the student, but rather is the outcome of an admission game played by the fraternity and students. A second central difference between our economy and this literature is that fraternity membership is not productivity enhancing, and uncertainty does not relate to whether or not an investment was made.

Austen-Smith and Fryer (2) consider a setting in which a “peer group” generates additional utility from leisure for its members, and this drives members to shift time allocation toward less education, which leads to lower wages. Those who are rejected by the peer group study more, “acting white”.

1.2 Fraternities and Sororities

To ease presentation, we drop gender differences and refer to both fraternities and sororities as “fraternities”. The first club-like fraternity with a centralized organization, was Kappa Alpha Society, founded in 1825. For a history and current status of fraternities, see Baird’s Manual (1).

Fraternities require pledge applicants to submit extensive information about themselves: their school GPA, recommendations, interests and useful skills. Fraternities devote far more time to evaluating applicants than do potential employers. In particular, almost all fraternity applicants are interviewed, and applicants take part in an extensive series of activities during the evaluation process. For example, Sigma Chi requires a potential member to spend one year working for the fraternity before the pledge. This suggests that fraternities are well-situated to evaluate a pledge applicant’s ability, so that fraternity membership can provide firms with valuable information.

Fraternities rely on membership fees and donations to fund activities. A substantial share of a fraternity’s income comes from alumni donations. Because high income alumni donate more, fraternities care about the future job market outcomes of members. An indication of the value that fraternities place on productive members is that GPA-based stipends to fraternity members are widespread. Fraternities frequently reject pledge applicants. Conversely, many highly-productive students choose not to apply to fraternities. Finally, students almost never join more than one fraternity. This reflects both secrecy issues (secret handshakes, for example, allow one member to verify the membership status of others), and because fraternity activities are quite time-consuming.
2 The Fraternity Game

There is a population of measure 1 of students. A student is fully described by his future employment productivity \( \theta \) and his fraternity socializing value, \( \mu \). Students have separable preferences over income and fraternity membership: a non-member’s payoff corresponds to his expected net lifetime income, \( M \), and a fraternity member with socializing value \( \mu \) derives utility

\[ M + n\mu, \]

where \( n > 0 \). \( M \) equals the student’s expected lifetime future wage minus the monetary value \( c \) of the time costs of fraternity service activities. Note that to sever all links with the club-good literature, we assume away any externalities from the socializing values of other fraternity members. So, too, to ensure that there is no direct link between productivity and membership, we assume that \( \theta \) and \( \mu \) are uncorrelated in the population. That is, the density over \( \theta \) and \( \mu \) is given by

\[ h(\theta, \mu) = h(\theta)h(\mu), \]

where the bounded supports of \( \theta \) and \( \mu \) are given by \( [\underline{\theta}, \bar{\theta}] \) and \( [\underline{\mu}, \bar{\mu}] \), respectively, and \( \underline{\theta} \) and \( \bar{\mu} \) are both positive. The associated cdf is \( H \), and the measure \( m \), used in some proofs, is based on \( H \). We emphasize that while we believe that socializing skills and productivity may be correlated in practice, we assume all correlation away in order to highlight the impact of application decisions by students and filtering by fraternities on the equilibrium distribution of abilities in the fraternity.\(^4\)

There is a single representative fraternity that chooses which “rush” applicants to admit. The fraternity cares about both the future market wages that its members will obtain, and the socializing values of its members. For simplicity, we assume that the fraternity has separable linear preferences over wages and socializing values, so that the fraternity’s payoff from members \((\theta, \mu)\) in the set \( C \) of fraternity members is given by

\[ \int_{(\theta, \mu) \in C} [W_1E_{\tilde{\theta}}(w(\tilde{\theta}|\theta)) + W_2\mu]h(\theta, \mu)d\theta d\mu, \]

where \( W_1, W_2 > 0 \). We assume that the fraternity is limited by space constraints to admitting at most a measure \( \Gamma \) of students: in practice a fraternity has a limited number of bedrooms in its build-

\(^4\)Obviously, if social skills and productivity are positively correlated in the population, and the fraternity values social skills, then this exogenous correlation will lead to fraternity members receiving higher wages than non-members; we wanted to avoid building this result trivially into our model. We assume away any network services that a fraternity might provide for the same reason.
ing. This means that the fraternity will tradeoff between $\mu$ and $\theta$ in admission—trading off future higher contributions from more able and hence wealthier alumni against their social contribution.

Our analysis is qualitatively unaffected by alternative preferences of the fraternity that continue to induce the fraternity to tradeoff between socializing value and ability in admissions. In particular, qualitatively identical outcomes obtain if the fraternity did not face a space constraint, but instead cared about the average socializing value of its members (say due to externalities), in addition to the future wages that members earn. So, too, outcomes are qualitatively unaffected if the fraternity, rather than facing a space constraint, incurred costs that were a convex function of the measure of members (say due to cramming more students into each room), or if the fraternity cared about the market value of the time contributions of its members.

After graduation, students are employed by firms. We assume that several risk-neutral firms make simultaneous wage offers to students. The firms do not observe an individual student’s productivity $\theta$ or fraternity socializing value $\mu$. However, firms do observe whether a student is a member of a fraternity. Firms also observe a common signal $\tilde{\theta}$ about the student’s productivity $\theta$, where $\tilde{\theta}$ is distributed according to $F_{\tilde{\theta}}(\cdot|\theta)$. We assume that more able students are more likely to generate higher signals: $F_{\tilde{\theta}}(\tilde{\theta}|\theta)$ is strictly decreasing in $\theta$ for all $(\theta, \tilde{\theta})$ with $F_{\tilde{\theta}}(\tilde{\theta}|\theta) \in (0,1)$.

There are four stages to our “fraternity rush” game. At stage one, each student type $(\theta, \mu)$ decides whether to apply for fraternity membership. We let $a(\theta, \mu)$ be an indicator function taking on the value 1 if student type $(\theta, \mu)$ applies, and taking on the value 0 if the student type does not apply. We sometimes use the set $A = \{(\theta, \mu)|a(\theta, \mu) = 1\}$. At stage 2, the fraternity chooses which applicants to accept. Off-equilibrium path characterizations are not intrinsically interesting, and to ease presentation, we characterize paths in which the set of students who make errors in their fraternity applications has measure zero. We let $b(\theta, \mu)$ be an indicator function taking on the value 1 if, along the equilibrium path, the fraternity would admit a student type $(\theta, \mu)$ who applied, and taking on the value 0 otherwise. We use $B$ to represent the set of admitted student types. Then the set of fraternity member types is $C = \{(\theta, \mu)|a(\theta, \mu)b(\theta, \mu) = 1\} = \{(\theta, \mu)|(\theta, \mu) \in A \cap B\}$, and the set of nonmembers is $\bar{C} = \{(\theta, \mu)|a(\theta, \mu)b(\theta, \mu) = 0\}$. At stage 3, firms see whether an individual is a fraternity member, and they see a noisy signal of his ability. Competition drives firms to offer that individual a wage equal to his expected productivity given that information. We let $\rho_F(\theta, \mu)$ denote firm beliefs about fraternity membership for each type $(\theta, \mu)$, where $\rho_F(\theta, \mu) = 1$ if firms
believe $(\theta, \mu)$ is a member of the fraternity, and $\rho_F(\theta, \mu) = 0$ if not. Finally, $w_C(\bar{\theta})$ denotes the wage of a fraternity member who emits the signal $\tilde{\theta}$, and $w_{C'}(\bar{\theta})$ denotes the wage of a non-member who generates signal $\bar{\theta}$. At stage 4, a worker with productivity $\theta$ produces output with value $\theta$.

An **equilibrium** is a collection $\{a(\theta, \mu), b(\theta, \mu), w_C(\bar{\theta}), w_{C'}(\bar{\theta})\}$ and firm beliefs $\rho_F(\theta, \mu)$ such that

i) $a(\theta, \mu) = 1$ if $E[w_C(\bar{\theta})|\theta] + n\mu - c \geq E[w_{C'}(\bar{\theta})|\theta]$; 0 otherwise.

ii) $b(\theta, \mu) = 1$ if $(\theta, \mu) \in B$, and $b(\theta, \mu) = 0$ if $(\theta, \mu) \notin B$, a.e. $(\theta, \mu)$, where $B$ solves

**Problem 1**

$$\max_B \int_{(\theta, \mu) \in A \cap B} [W_1 E[w_C(\bar{\theta})|\theta] + W_2 \mu] h(\theta, \mu) d\theta d\mu$$

subject to $m(A \cap B) \leq \Gamma$.

iii) wages are competitive given beliefs by firms $\rho_F(\theta, \mu)$:

$$w_C(\bar{\theta}) = \frac{\int_C \theta h(\theta, \mu) \rho_F(\theta, \mu) f_{\bar{\theta}}(\bar{\theta}|\theta) d\theta d\mu}{\int_C h(\theta, \mu) \rho_F(\theta, \mu) f_{\bar{\theta}}(\bar{\theta}|\theta) d\theta d\mu}; \quad w_{C'}(\bar{\theta}) = \frac{\int_C \theta h(\theta, \mu) (1 - \rho_F(\theta, \mu)) f_{\bar{\theta}}(\bar{\theta}|\theta) d\theta d\mu}{\int_C h(\theta, \mu) (1 - \rho_F(\theta, \mu)) f_{\bar{\theta}}(\bar{\theta}|\theta) d\theta d\mu}.$$

iv) Beliefs are consistent: For a.e. $(\theta, \mu)$, $\rho_F(\theta, \mu) = a(\theta, \mu)b(\theta, \mu)$.

Measure-zero perturbations of a set $B$ that corresponds to some equilibrium are also an equilibrium. These measure zero perturbations are uninteresting. Accordingly we focus on a best response of the fraternity that is a **good set**, i.e., a set $B$ that is equal to the closure of its own interior.

We begin by providing conditions under which fraternity membership has no effect on labor market outcomes.

**Theorem 1** Suppose that firms either receive perfect signals about students, i.e. $\tilde{\theta} = \theta$, a.e., $\theta$, or firms receive perfectly uninformative signals, $F_{\tilde{\theta}}(\cdot|\theta) = F_{\tilde{\theta}}(\cdot|\theta')$, for all $\theta, \theta'$. Then, in equilibrium, a student’s wage does not depend on whether he is in the fraternity or not, i.e., $w_C(\bar{\theta}) = w_{C'}(\bar{\theta})$, for all $\bar{\theta}$. Hence, a student type $(\theta, \mu)$ applies for membership in the fraternity if and only if $n\mu - c \geq 0$.

Suppose signaling is perfect. Then $w_C(\bar{\theta}) = w_C(\bar{\theta}) = \bar{\theta} = \theta$ (a.e.). Optimization by students then immediately implies that a student type $(\theta, \mu)$ applies if and only if $n\mu - c \geq 0$, independently of
θ. In contrast to students, the fraternity selectively admits higher θ applicants who will earn higher wages. In particular, the fraternity trades off between μ and θ in admission; letting μ_B(θ) denote the boundary of the admission set, indifference implies that the boundary has slope \( \frac{d\mu(\theta)}{d\theta} = -\frac{W_1}{W_2} \).

Figure 1 illustrates the equilibrium. The solid line is the fraternity’s equilibrium cutoff rule—all types to the right of the line who apply are accepted, while all those to the left are rejected. The vertical dashed line represents the accept-or-reject line of students—student types to the right apply in equilibrium, i.e., are in the set A. Hence, the equilibrium set \( C \) of fraternity members consists of those types to the right of both the dashed and solid lines, and the measure of the set \( C \) is at most \( \Gamma \).

If, instead, signals are completely uninformative, then all individuals receive the same wage, \( w_C(\tilde{\theta}) = w_C(\theta) = E[\theta] \). A fraternity would like to commit to excluding low \( \theta \) students with high socializing values \( \mu \), and to accepting high \( \theta \) students with low socializing values. However, since firms have no source other than fraternity membership for evaluating a student’s ability, all fraternity members must receive the same wage. But then, given any beliefs that firms hold about the abilities of fraternity members, the fraternity’s optimal admission policy only depends on \( \mu \), admitting a type \((\theta, \mu)\) if and only if \( \mu \) exceeds some critical cutoff. Hence, fraternity membership conveys no information to firms about \( \theta \). Hence, in equilibrium, both fraternity and non-fraternity
members receive wage $E[\theta]$. Since wages do not depend on membership, it follows that only students with $n\mu \geq c$ apply. Figure 2 illustrates this uninformative signal case.

\[\text{Figure 2: Contentless signaling equilibrium.}\]

Although we do not explore it further, the case of completely uninformative signals about abilities highlights the gains that fraternities may achieve from an ability to commit to their admission policies. In particular, the fraternity would like to commit to excluding low ability students who have moderately high socializing values, and to accepting high ability students with lower socializing values. In practice, imperfect commitment devices that fraternities use include having university officials report the average GPA of members, and having the Greek council forbid fraternity participation to students with GPAs below some standard. This commitment induces a fraternity to weigh ability in admission, raising wages of members, and thereby raising the fraternity’s payoff.

The central implication of Theorem 1 is that for fraternity membership to affect job market outcomes, firms must receive signals about student productivities that are noisy, but not perfectly so. Then, because more productive students tend to earn higher wages, fraternities value both productivity and fraternity socializing value, and will tradeoff between the two in admission. We now examine the choice problems of students and the fraternity in more detail.
2.1 Student’s Problem

Students compare the expected payoffs from being a fraternity member and not, taking into account both the consequences for expected wages, and his fraternity socializing value. Optimization immediately implies that a student type \((\theta, \mu)\) applies for fraternity membership if and only if
\[
E_{\tilde{\theta}} \left[ w_C \left( \tilde{\theta} \right) \mid \theta \right] + n\mu - c \geq E_{\tilde{\theta}} \left[ w_{\bar{C}} \left( \tilde{\theta} \right) \mid \theta \right].
\]
(2)

That is, a student applies to the fraternity either to obtain higher expected wages, or because his fraternity socializing value \(\mu\) is sufficiently high. In particular, if (2) holds, then student type \((\theta, \mu)\) applies even if he expects to be rejected by the fraternity. The following is immediate.

**Theorem 2** If \(a(\theta, \mu) = 1\), then \(a(\theta, \mu') = 1\), for all \(\mu' > \mu\).

**Proof.** The expected wages of individuals \((\theta, \mu)\) and \((\theta, \mu')\) are the same, but a type \((\theta, \mu')\) student gains strictly more utility from joining the membership. ■

**Corollary 1** The equilibrium supply of fraternity applicants is summarized by a continuous function \(\mu_A(\theta)\) such that a type \((\theta, \mu)\) student applies if and only if \(\mu \geq \mu_A(\theta)\).

**Proof.** Immediate. The expected wages of fraternity and non-fraternity members are continuous in \(\theta\); so that student payoffs and hence choices are continuous in expected wages. ■

2.2 Fraternity’s Problem

In any equilibrium, the fraternity offers membership to the set of students \(B\) that solves Problem 1. The sets \(A\) and \(B\) implicitly define the set of fraternity members \(C = A \cap B\) and the set of nonmembers \(\bar{C}\). Since the fraternity’s payoff is increasing in the socializing values of its members, we have

**Theorem 3** For almost all \((\mu, \theta)\) in \(A \cap B\), almost all types \((\theta, \mu')\) with \(\mu' > \mu\) also belong to \(B\).

**Proof.** See the appendix. ■

We next show that we can extend this characterization to establish that the fraternity also wants to admit students who are more able as long as expected wages are increasing in \(\theta\); and expected wages are increasing in \(\theta\) if \(w_C(\tilde{\theta})\) is increasing in \(\tilde{\theta}\). As a preliminary step, we present an implication of the MLRP property on signals (see Milgrom (7)) for wages in any equilibrium.

10
Lemma 1 Assume that $f(\theta|\tilde{\theta}) > 0$ on $[\theta, \bar{\theta}]$, and that $f(\theta|x)$ satisfies the MLRP property. Then for any subset $C \subset \Theta \times \tilde{\Theta}$ with $P(\tilde{\theta} = k|\tilde{\theta} \in C) < 1$ for all signals $k$, $E_{\tilde{\theta}}[E(\theta|\tilde{\theta})|\theta, C]$ increases with $\theta$.

Proof. See the appendix. ■

Theorem 4 Suppose that the signals that firms receive about student abilities have the MLRP property. Then for almost all $(\theta, \mu)$ if $b(\theta, \mu) = 1$, we have $b(\theta', \mu) = 1$ for almost all $\theta' > \theta$.

Proof. Immediate. By Lemma 1, the expected wage $E_{\tilde{\theta}}[w_C(\tilde{\theta})|\theta]$ is an increasing function of $\theta$. The logical construction of Theorem 3 then applies. ■

Theorems 3 and 4 pin down the attributes of the set $B$ of student types that the fraternity would admit. For example, if every student whom the fraternity would want to admit applies, then $B$ is defined by a negatively-sloped curve in $(\theta, \mu)$ space, $\mu_B(\theta)$: The fraternity admits almost every student type above (Theorem 4) and to the right (Theorem 3) of this curve (see Figure 3), i.e., $B = \{(\theta, \mu)|\mu \geq \mu_B(\theta)\}$, and $C = A \cap B$. Both $\mu_A(\theta)$ and $\mu_B(\theta)$ are continuous in $\theta$, reflecting the continuity of expected wages in $\theta$.

More generally, for almost all $\theta$ where the fraternity’s admission decision is not constrained by student application, i.e., for almost all $\theta$ with $\mu_B(\theta) > \mu_A(\theta)$, the fraternity trades off linearly between expected wage and fraternity socializing value in admission. That is, for $\theta_1, \theta_2$ with
\( \mu_B(\theta_j) > \mu_A(\theta_j), \ j = 1, 2, \) we have

\[
W_1E(w_C(\bar{\theta})|\theta_1) + W_2\mu(\theta_1) = W_1E(w_C(\bar{\theta})|\theta_2) + W_2\mu(\theta_2).
\]

That is, marginal contributions of these marginal types, \((\theta_1, \mu_B(\theta_1))\) and \((\theta_2, \mu_B(\theta_2))\) are equal.\(^5\)

2.3 Existence of equilibrium

We first characterize when the “empty fraternity” is an equilibrium. In this “Groucho Marx” equilibrium, the fraternity would accept anyone who applies, but no one applies because firms believe that anyone who joins the fraternity has low ability \(\bar{\theta}\) and hence would be given wage \(w_C(\bar{\theta}) = \bar{\theta}\). If no one is a member of the fraternity, then someone who generates signal \(\tilde{\theta}\) receives wage \(w_C(\tilde{\theta}) = E[\theta|\tilde{\theta}]\). Let \(w = E_{\tilde{\theta}}[w_C(\tilde{\theta})|\tilde{\theta}]\) be the wage that a student with lowest ability \(\theta\) expects if he does not join the fraternity in this scenario.

**Theorem 5** Suppose that the signaling technology has a full support property, \(f(\theta|x) > 0, \forall x\). Then an equilibrium exists with \(A = C = \emptyset\) if and only if \(n\bar{\mu} - c \leq w - \bar{\theta}\).

**Proof.** See the appendix. \(\blacksquare\)

If \(n\bar{\mu} - c \leq w - \bar{\theta}\), then pessimistic firm beliefs can support the empty fraternity equilibrium. However, if the inequality does not hold, then sufficiently inept students with high socializing values would prefer to join the fraternity because they also expect to receive low enough wages outside the fraternity that the maximum wage cost from joining the fraternity is more than offset by their high socializing values.

We next prove that an equilibrium always exists to this fraternity game, establishing a fixed point to a mapping from conjectured optimal student application and fraternity admission choices by firms to the best responses to those conjectures by students and fraternities. To do so, we exploit Theorems 2 and 4 and consider continuous student and fraternity choice functions \(\mu_A(\cdot)\) and \(\mu_B(\cdot)\), where a student type \((\theta, \mu)\) is a member of the fraternity if and only if \(\mu \geq \max\{\mu_A(\theta), \mu_B(\theta)\}\).

Existence of equilibrium then follows from standard fixed point theorems.

\(^5\)This result extends if we relax the structure on the fraternity’s preferences, so that preferences over aggregate wages and socializing values are non-linear, \(W(m(Ew_C(\bar{\theta})), m(C))\). Then, the appropriate marginal derivatives, \(W_1, W_2\), evaluated at the aggregates, describe the indifference relationship for the fraternity.
Theorem 6  An equilibrium exists to the fraternity game.

Proof. See the appendix. ■

3 Three signal economy

To gain insight into the equilibria of this fraternity game, we now focus on an economy in which student productivities and fraternity socializing values are uniformly distributed on the unit square, i.e., \((\theta, \mu)\) are uniformly distributed on \([0; 1] \times [0; 1]\), and students generate one of three possible signals, \(\tilde{\theta} \in \{H, M, L\}\). In particular, we suppose that more able students with \(\theta > 0.5\) generate either medium or high signals, where the probability of a high signal is linearly increasing in ability; and that less able students with \(\theta < 0.5\) generate either low or medium signals:

\[
\begin{align*}
\Pr(H|\theta) &= (2\theta - 1), \quad \theta > \frac{1}{2} \quad \text{and 0 otherwise.} \\
\Pr(L|\theta) &= (1 - 2\theta), \quad \theta < \frac{1}{2} \quad \text{and 0 otherwise.} \\
\Pr(M|\theta) &= 1 - \Pr(L|\theta) - \Pr(H|\theta).
\end{align*}
\]

This signal technology obviously satisfies the MLRP property. Its central feature is that a student with \(\theta < 0.5\) hopes to get lucky and receive a medium signal, and thereby be indistinguishable from a student with \(\theta > 0.5\) who unluckily receives a medium signal.

Let \(w_C(\tilde{\theta})\) be the wage that a fraternity member who generates signal \(\tilde{\theta}\) receives and \(w_C(\tilde{\theta})\) be the wage that a non-member who generates signal \(\tilde{\theta}\). The expected wage of a student with ability \(\theta\) who joins the fraternity is

\[
E \left( w_C(\tilde{\theta}) \mid \theta \right) = w_C(H) \Pr(H|\theta) + w_C(M) \Pr(M|\theta) + w_C(L) \Pr(L|\theta).
\]

An analogous expression describes wages of students who are not fraternity members.

The piecewise linear structure of the signaling technology implies that the expected wage functions are piecewise linear in \(\theta\) with a single kink at \(\theta = \frac{1}{2}\). It follows that the boundary describing the set of students that the fraternity would admit, where not limited by students’ application decisions, is also linear with a kink at \(\theta = \frac{1}{2}\). Since the difference in wages of fraternity members and non-members is linear with a kink at \(\theta = \frac{1}{2}\), the boundary of the set of applicants to the
fraternity, $\mu_A(\theta)$, is also linear with a kink at $\theta = \frac{1}{2}$. Therefore, the set of fraternity members, $\{ (\mu, \theta) | \mu \geq \max\{ \mu_A(\theta), \mu_B(\theta) \} \}$, is described by a continuous piecewise-linear function from $[0, 1]$ to $[0, 1]$ that has one or two kinks, where one kink is at $\theta = 0.5$, and the other (if it exists) is at the intersection of the fraternity and student cutoff rules.

One equilibrium is obviously the “empty fraternity”, but there are also more interesting equilibria. In particular, given $\Gamma$, we search for (i) an equilibrium in which the boundary $\mu_A(\theta)$ of $A$ is everywhere to the left of the boundary $\mu_B(\theta)$ of $B$, i.e., where every student that the fraternity would want is admitted and (ii) an equilibrium in which $\mu_A(\theta)$ and $\mu_B(\theta)$ intersect, so the piecewise-linear function describing the frontier of the set of fraternity members has two kinks. This latter equilibrium is described by a system with thirteen unknowns (the slope and intercepts of the three lines plus the intersection point of the student and fraternity frontier, plus six wages) and thirteen equations (6 equations from the firm’s problem — wages equal expected skill given signal realization and membership status, 4 equations from the fraternity and 3 equations from the students). We solve this system numerically for the unique associated equilibrium outcome, when it exists.

![Figure 4: Application-unconstrained equilibrium.](image)

In our base parameterization, student utilities are $M + n\mu$, with $n = 0.18$, student time costs of participating in the fraternity are $c = 0.09$, the fraternity trades off between wages and socializing value according to $\frac{W_1}{W_2} = 1.1$, and the fraternity’s capacity is $\Gamma = 0.35$. Figure 4 illustrates the
unique “application-unconstrained” equilibrium. In this equilibrium firms have optimistic beliefs about the productivities of fraternity members, so that given any signal emitted by a student, his wage is higher if he is a member of a fraternity than if he is a non-member. As a result, in this equilibrium every student that the fraternity would like to admit chooses apply—and, indeed, because $w_C(H) = 0.8480 > w_{\pi\pi}(H) = 0.8143$, only very productive people with especially low socializing values choose not to apply (and while less productive students apply, most are rejected).

However, for exactly the same parameterization, there is also an “application constrained” equilibrium in which some students whom the fraternity would like to admit do not apply. Figure 5 depicts this equilibrium: the solid line denotes the fraternity’s cutoff rule, and dashed line denotes locus of students who are indifferent between joining the fraternity and not. In this equilibrium, firms hold more pessimistic beliefs about the abilities of fraternity members, so that higher ability students are more reluctant to join the fraternity. Intermediate quality students remain eager to join, and the fraternity’s composition is radically shifted to reflect this population. Comparing the fraternity’s cutoff line in Figure 4 with that in Figure 5 reveals that the fraternity is less “picky” when its choice set is constrained by the reluctance of able students to apply. Because able types $\theta = 1$ expect lower wages inside the fraternity than out, $w_C(H) = 0.7940 < w_{\pi\pi}(H) = 0.8555$, the fraternity attracts only a small fraction of able students, and the bulk of its members have intermediate abilities.
Figure 6: Equilibrium Wages. The solid line is the expected wage of a fraternity member (as a function of $\theta$), and the dashed line is the expected wage of a nonmember.

Figure 6 presents the expected wage that a student with ability $\theta$ would receive as a fraternity member and non-member for these two equilibria. Notice the crossing of wages in the application-constrained equilibrium. This reflects that while all lower ability student receive higher wages as fraternity members, higher $\theta$ students in the application-constrained equilibrium accept a direct loss in wage by joining the fraternity, for which they are compensated by high socializing values.

Note that in the application-unconstrained equilibrium, were we to increase $\Gamma$ slightly, then all existing members of the fraternity would still apply, and the fraternity’s payoff would be increased. This observation implies that were we to replace the fraternity’s capacity constraint with a strictly convex cost function of admitting more members, the fraternity would admit more members when beliefs of firms about member abilities are optimistic, thereby encouraging able students to apply.

Figure 7 reveals how the fraternity’s capacity affects equilibrium outcomes. Interestingly, raising capacity can raise the wages of able fraternity members. Essentially, when $\Gamma$ is increased, the mix of students that the fraternity admits shifts slightly toward more able students with lower socializing values, i.e., toward students with higher $\theta$s and lower values of $\mu$. But this raises the expected wages of able students who join the fraternity. But then, able students are more willing to join—there is a significant increase in the measure of able students who apply to the fraternity. Notice also that as $\Gamma$ increases, the slope of the boundary characterizing the application decision of less able students with $\theta \in [0, \frac{1}{2}]$ changes. This result reflects a change in the relative slope of wage functions:
when, among students with $\theta \in [0, \frac{1}{2}]$, most of those with relatively high productivities are in the fraternity, then receiving the signal $M$ and being outside the fraternity has a smaller premium than being in the fraternity and getting signal $M$ (relative to receiving signal $L$ in both cases).

### 3.1 Empirical Analysis

To investigate the extent to which our model is consistent with the actual application and selection process of fraternities, we obtained data on the cumulative GPAs of the 8634 seniors at the University of Illinois in the fall semester of 2007 (excluding international students on temporary visas), and a random sampling of 701 seniors who were fraternity or sorority members. GPAs only reflect courses taken at the University of Illinois (omitting transferred courses), but the senior classification is based on all hours accumulated prior to the end of the fall, 2007 term. One can interpret a student’s GPA as a noisy indicator of his or her ability. Figure 8 presents the conditional probability that a student is a member of a fraternity given his or her GPA. Figure 8 reveals that the conditional probability that a student with a low GPA of 2.0 is a fraternity member is less than 0.05, but that this probability more than triples for intermediate GPAs between 3 and 3.4, before falling by more than a third for students with high GPAs. This inverted U-shaped pattern is
Figure 8: Prob(fraternity member|GPA)

Notes: The dashed line is the conditional probability that a student is a fraternity member given his or her GPA (rounded to the nearest 0.2). The thin line graphs the distribution of GPAs for 701 seniors who are fraternity members (fall 2007), and the thick line graphs the probability distribution for all 8634 seniors at the University of Illinois (fall 2007). The bars indicate 2 standard deviation confidence intervals.

precisely what emerges in the equilibrium to our fraternity game where able students are reluctant to join fraternities to avoid being tainted in labor market outcomes, while intermediate and less able students are eager to join, and the fraternity screens out most of the less able students, i.e., those who do not have high socializing values.

This qualitative finding suggested that it was plausible to estimate our three-signal model formally. Because our underlying model has uniformly distributed abilities, we assume that $\theta$ corresponds to the quantile of the GPA distribution (so that $\theta$ is distributed uniformly); and that $\mu$ is distributed uniformly. Letting $\Phi$ be an event that a random person is in the fraternity, the
fraction of students with ability $\theta$ who are in the fraternity is

$$1 - \max(\mu_A(\theta), \mu_B(\theta)) = P(\Phi|\theta) = \Pr(\Phi) \frac{f_{\theta}(\theta|\Phi)}{f_{\theta}(\theta)},$$

where $\mu_A(\theta)$ and $\mu_B(\theta)$ are the cutoff rules of students and fraternity, $f_{\theta}(\cdot)$ is the density of $\theta$, and $\Pr(\Phi)$ is a probability that a random senior is a member of the fraternity. The densities are estimated using a kernel estimator. To estimate $f_{\theta}(\cdot)$ we use the sample of all senior students with GPAs of at least 2,\(^6\), and we use the sample of fraternity members’ GPAs exceeding 2 to estimate $f_{\theta}(\cdot|\Phi)$; and our estimate of $\Pr(\Phi)$ is $\frac{1345}{8634}$, the number of senior fraternity members, divided by the number of seniors.

Next, we take 20 equally spaced $\theta$s between 0.05 and 0.95 as our pseudosample, and evaluate $\max(\mu_A(\theta), \mu_B(\theta))$ at these points. They are represented by dots in Figure 9.

The cutoff line in our model has two kinks, one at $\theta = 0.5$, and another that we estimate. We first do this non-structurally, and then contrast the fit with that obtaining when we penalize cut-off rules that are inconsistent with equilibrium. Our non-structural approach uses a two-step estimation procedure to estimate the kink. We first take a possible kink value as given, and find the slopes of the cut-off rules that minimize the SSE; we then determine the location of the kink that minimizes the SSE overall.

\[\text{Figure 9: Estimated Structural Model}\]

\begin{notes}
This displays the unconstrained model estimates and the penalized estimates.
\end{notes}

Direct structural estimation is complicated by the extreme nonlinearity of the equilibrium re-

\(^6\)We drop the few students with GPAs below 2, as they are subject to screening by the University (and, indeed, only students with GPAs of at least 2 can graduate).
quirement. This leads us to adopt a lasso-type estimation approach, in which we minimize the residual sum of squares plus a quadratic measure of the distance from the equilibrium. In equilibrium, \( \frac{W_1}{W_2} = \frac{b_1}{2(w_C(H) - w_C(M))} = \frac{b_2}{2(w_C(M) - w_C(L))} \), where \( b_1 \) is the slope of the club’s cutoff rule below \( \theta = 0.5 \), and \( b_2 \) is the slope for \( \theta > 0.5 \). We use the penalty function

\[
10 \left[ b_1 \left( w_C(H) - w_C(M) \right) - b_2 \left( w_C(M) - w_C(L) \right) \right]^2.
\]

The first panel of Figure 9 presents the estimated cut-off rules from unconstrained estimation approach. This fit is far from an equilibrium; most obviously, the cut-off rule for the club is not a monotonically decreasing function of \( \theta \). The panel on the right presents the estimated cut-off rules from the penalized estimation approach.\(^7\) An F-test\(^8\) indicates that the differences are not statistically significant.

The penalized estimation approach implies estimates of the primitives for students: the cost \( c \) is 0.23, or about 46% of the unconditional expected wage, and the fraternity socializing parameter is \( n = 0.28 \). This means that students with \( \mu > \frac{c}{n} \approx 0.82 \) obtain a net utility benefit from joining the fraternity. Figure 9 shows that most fraternity members are above this threshold. We also see that relative to a full information setting, the wage-setting mechanism impedes the efficiency of club participation: there are too many fraternity members with intermediate abilities and too few low and high ability students with high socializing values. The club’s relative weighting on member wages versus socializing values, \( \frac{W_1}{W_2} \), is 0.22. We bootstrap the estimator to obtain 95% confidence intervals. These confidence intervals provide some interesting insights. First, the fraternity’s capacity is precisely estimated (95% confidence interval of \([15.44\%, 15.76\%] \)), but estimates of \( n \) and \( c \) are not (confidence intervals are \([0.11, 0.52]\) and \([0.08, 0.44]\), respectively). The fraternity’s relative weighting on member wages to socializing values also has a wide confidence interval, \([0.06, 0.33]\). However, the ratio of \( \frac{c}{n} \), the threshold of efficient club membership, is stable, with a tight 95% confidence interval of \([0.73, 0.84]\).

\(^7\)The value of the penalty is less than \(10^{-10}\), indicating that the estimated model is very close to an equilibrium model.

\(^8\)The F-value is 2.93, (1,13) degrees of freedom, p-value of 0.11; we are omitting considerations of the non-normality of errors and the nonlinearity of both the model and restrictions. The asymptotic distribution of this test is \(\chi^2(1)\), with a p-value of 0.08.

20
4 Conclusion

On first impression, it is not clear why fraternity or sorority membership should matter for labor market outcomes—fraternity activities seem to have little to do with skill development for future careers. Nonetheless, resumes regularly highlight fraternity membership, suggesting that membership augments the other signals that employers use to evaluate a person’s productivity. Our paper provides insights into when fraternity membership matters for labor market outcomes. We first show that if firms can either evaluate student productivities perfectly, or are completely incapable of screening job applicants, then fraternity membership has no impact on labor market outcomes. Otherwise, fraternity membership matters. In particular, we identify two equilibria in which fraternity membership is valued by some students for labor market outcomes. In one equilibrium, optimistic beliefs by firms about the abilities of fraternity members lead to higher wages for fraternity members than non-members. As a result, everyone whom the fraternity would like to admit chooses to pledge. We also identify an equilibrium in which able students are harmed in the labor market by fraternity membership, but less able students benefit. In this equilibrium, most fraternity members have intermediate abilities: less able students apply, hoping to be mixed in with better students, but are rejected unless they have high fraternity socializing values, while very able students who lack high socializing values do not apply to avoid being tainted in labor market outcomes due to being mixed in with less able fraternity members. We find that this latter equilibrium can reconcile the qualitative features of the ability distributions of fraternity members and non-members at the University of Illinois.

While we pose our analysis in the context of fraternities, the central economic story extends with some variations to filtering by other organizations. For example, ROTC (reserve officer training corps) may value both intellectual ability and leadership skills that firms value, but also physical fitness that does not contribute productively in many occupations. As a result, even were ROTC not to directly build skills of its officers, our model indicates that firms may rationally weigh ROTC membership positively in their evaluations of job-seekers.
5 Appendix

Proof of Theorem 3: First observe that in light of Theorem 2, if $(\theta, \mu)$ applies to the fraternity in equilibrium, then so does $(\theta, \mu')$. Suppose the theorem were false. Then for $\varepsilon > 0$, sufficiently small, the set

$$\Theta_\varepsilon = \{\theta \exists \theta_0 : \int_{-\infty}^{\theta_0} a(\theta, x)b(\theta, x)h_\mu(x)dx > \varepsilon, \int_{\theta_0}^{+\infty} (1 - b(\theta, x))h_\mu(x)dx > \varepsilon\}$$

has positive measure, i.e., there exists $\delta > 0$ such that $\int_{\Theta_\varepsilon} h_\theta(x)dx > \delta$. For every $\theta$ in $\Theta_\varepsilon$ pick a set of $K = \{(\theta, \mu)|\mu < z_\theta, a(\mu, \theta)b(\mu, \theta) = 1\}$ and $L = \{(\theta, \mu)|\mu > z_\theta, b(\mu, \theta) = 0\}$ such that $\int_K a(\theta, \mu)b(\theta, \mu)h_\mu(x)dx = \int_L a(\theta, \mu)b(\theta, \mu)h_\mu(x)dx = \varepsilon$. But then the fraternity decision rule

$$\hat{b}(\theta, \mu) = b(\theta, \mu)(1 - I((\theta, \mu \in K)) + I((\theta, \mu \in L))$$

strictly raises the fraternity’s payoff as $E(\mu|K) < E(\mu|L)$ and the expected wages generated by members and fraternity size are unchanged. Therefore, $b$ could not have been an equilibrium strategy for the fraternity. □

Proof of Lemma 1: Consider two signals, $x > y \in \Theta$, and two productivities, $\theta_2 > \theta_1 \in \Theta$, such that $(x, \theta_1), (x, \theta_2)$ and $(y, \theta_2) \in C$. By the MLRP property,

$$\frac{f(\theta_2|x)}{f(\theta_1|x)} > \frac{f(\theta_2|y)}{f(\theta_1|y)}.$$

Notice that for every $(j, k) \in C$, $f(j|k, C) = \frac{f(j|k, (j,k) \in C)}{\int_{\Theta} f((j,k) \in C)dF(\theta|k)} = \frac{f(j|k)I((j,k) \in C)}{P(C)}$. Rewrite the MLRP condition:

$$\frac{f(\theta_2|x, C)}{f(\theta_1|x, C)} = \frac{f(\theta_2|x, I((\theta_2, x) \in C))}{P(C)} = \frac{f(\theta_2|x)}{f(\theta_1|x)} \geq \frac{f(\theta_2|y)}{f(\theta_1|y)} = \frac{f(\theta_2|y, C)}{f(\theta_1|y, C)}.$$

Therefore, if the MLRP condition holds for the entire support, it holds for a subset $C$ of that support. This condition ensures that $E(\theta|\tilde{\theta}, C)$ is an increasing function of $\tilde{\theta}$: By $F(x|\theta_2) \geq F(x|\theta_1)$,

$$E_\tilde{\theta}[E(\theta|\tilde{\theta})|\theta_2] = \int_{\tilde{\Theta}} E(\theta|\tilde{\theta})dF(\tilde{\theta}|\theta_2) > \int_{\tilde{\Theta}} E(\theta|\tilde{\theta})dF(\tilde{\theta}|\theta_1) = E_\tilde{\theta}[E(\theta|\tilde{\theta})|\theta_1].$$ □

Proof of Theorem 5: We first show that if $n\tilde{\mu} - c \geq \overline{w} - \underline{w}$, then the empty fraternity cannot be an equilibrium. If no one joins the fraternity, then equilibrium demands that $\underline{w}$ expect wage $\overline{w}$ if he does not join; and the expected wages of students with ability greater than $\underline{\theta}$ who do not
join exceed $w$. With the full support assumption, following any signal realization $\tilde{\theta}$, firms can hold equilibrium beliefs that the anyone who joins the fraternity and generated that signal has ability $\theta$. These beliefs minimize the wage of any student who joins the fraternity. Given these beliefs, since expected wages are continuous in $\theta$, if $n\tilde{\mu} - c + \theta > w$, then all students in a sufficiently small neighborhood of $(\theta, \tilde{\mu})$ would apply for fraternity membership, and since their measure is less than $\Gamma$, the fraternity would accept them. Hence, the empty fraternity cannot be an equilibrium.

Conversely, if $n\tilde{\mu} - c \leq w - \theta$, then given the pessimistic beliefs by firms, $w_C(\tilde{\theta}) = \theta$ so that $(\theta, \tilde{\mu})$ at least weakly prefers not to apply to the fraternity; and all other types strictly prefer not to apply. Hence, no one applying to the fraternity is an equilibrium. □

**Proof of Theorem 6:** To prove existence, it suffices to characterize student and fraternity choices via the continuous functions $\mu_A(\cdot)$ and $\mu_B(\cdot)$ (see Theorems 2 and 4), proving the existence of an equilibrium in which a student type $(\theta, \mu)$ is a member of the fraternity if and only if $\mu \geq \max\{\mu_A(\theta), \mu_B(\theta)\}$. In particular, given $w_C(\cdot)$ and $w_C(\cdot)$, $\mu_A(\theta)$ solves equation (2) at equality, for $\mu_A(\theta) \in (\tilde{\mu}, \mu)$. Since $\mu_A(\cdot)$ is uniquely defined, it follows that $\mu_B(\cdot)$ is uniquely defined. We have established that $\mu_j : [\theta, \tilde{\theta}] \rightarrow [\tilde{\mu}, \mu], j = A, B$, is continuous. The space of such functions, endowed with the weak$^*$ topology, is compact. So, too, we can focus on beliefs by firms about which student types are fraternity members that are summarized by continuous functions $\hat{\mu}_A(\cdot)$ and $\hat{\mu}_B(\cdot)$ about which student types apply and which ones are accepted by the fraternity, where $(\theta, \mu)$ is a conjectured fraternity member if and only if $\mu \geq \max\{\hat{\mu}_A(\theta), \hat{\mu}_B(\theta)\}$. These beliefs, $\hat{\mu}_A(\cdot), \hat{\mu}_B(\cdot)$ then determine competitive wage functions,

$$(\hat{w}_C(\tilde{\theta}), \hat{w}_C(\tilde{\theta})) = (E[\theta | \tilde{\theta}, \mu \geq \max\{\hat{\mu}_A(\theta), \hat{\mu}_B(\theta)\}], E[\theta | \tilde{\theta}, \mu < \max\{\hat{\mu}_A(\theta), \hat{\mu}_B(\theta)\}],$$

and these wage functions, in turn, imply optimal student and fraternity best response choices, $\mu_A(\cdot), \mu_B(\cdot)$. Hence, we have a mapping from $(\hat{\mu}_A(\theta), \hat{\mu}_B(\theta))$ to $(\mu_A(\theta), \mu_B(\theta))$. Equilibrium is given by a fixed point to this mapping from [conjectured by firms] optimal student and fraternity choices to the best response optimal student and fraternity choices; and we have just established that this mapping satisfies the conditions of Kakutani’s fixed point theorem. □
References


